Math 21b: First Midterm

Please show your work for all problems except for the True/False questions on the last page. If you need additional space, feel free to use the backs of the pages (please make a note if you do so, so we know to look). Each of the four questions will be worth ten points. No calculators, notes, books, or any other aids are allowed.

Please don't write on this front page (except for your name) as we will use it to record grades.

Name:

#1	
#2	
#3	
#4	
Total	

1. For an arbitrary constant k, consider the linear system

$$\begin{aligned} x - 2kz &= 0 \\ x + 2y + 6z &= 2 \\ 2z - kx &= 1 \end{aligned}$$

of equations in x, y, z.

(a) [4 pts.] For $k = 2$ the system has a unique solution. Find it, and check that it is actually a solution.

(b) [4 pts.] For which value(s) of k does the system *not* have a unique solution?

(c) [2 pts.] For the value(s) of k for which the system does not have a unique solution, which if any yield a system with no solution, and which if any yield a system with infinitely many solutions?

2. (a) [4 pts.] Compute the row-reduced echelon form (rref) of the matrix

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right].$$

(b) [4 pts.] Your computation in (a) gives the inverse of a 3×3 matrix A. What is A, and what is A^{-1}?

(c) [2 pts.] Use the matrices A and A^{-1} of (b) to solve the linear system represented by $A\vec{x} = \vec{b}$, where

$$\vec{b} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}.$$

Check your work by verifying that the entries of $\vec{x}$ actually satisfy that linear system.

3. Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ be the following vectors in $\mathbb{R}^4$:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 6 \\ 7 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \\ 1 \end{bmatrix}.$$

Let V be the linear subspace of $\mathbb{R}^4$ spanned by $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$.

(a) [2 pts.] Write a matrix whose image is V.

(b) [5 pts.] Find a basis of V.

(c) [3 pts.] Is the vector $\begin{bmatrix} -1 \\ 4 \\ 3 \\ 2 \end{bmatrix}$ in V?

4. For each of the following 10 assertions, circle **T** if the assertion is <u>true</u>, and circle **F** if the assertion is <u>false</u>. Each is worth one point. For this question only, there is no need to justify your answers.

T **F** a) If the linear system $A\vec{x} = \vec{b}$ has a unique solution then A must be a square matrix.

T **F** b) Reflection about the line $x + y = 1$ is a linear transformation of $\mathbb{R}^2$.

T **F** c) A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ always has infinitely many vectors in the kernel if $n > m$.

T **F** d) If A is a matrix such that $AAAAA = I_2$ then A is invertible.

T **F** e) If A is an invertible matrix and B is a matrix such that AB is a zero matrix, then B is a zero matrix.

T **F** f) If A and B are invertible $n \times n$ matrices then $(A + B)(A - B) = A^2 - B^2$.

T **F** g) Suppose a linear system has coefficient matrix A and augmented matrix B. If the system is consistent then $\text{rank}(A) = \text{rank}(B)$.

T **F** h) If the vector $\vec{u}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$, then $\vec{w}$ must be a linear combination of vectors $\vec{u}$ and $\vec{v}$.

T **F** i) If vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ are linearly independent, then vectors $\vec{v}_2, \vec{v}_3, \vec{v}_4$ must be linearly independent as well.

T **F** j) The set $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \text{ is an integer} \right\}$ (infinitely many vertical lines, see picture below) is closed under vector addition.