MATH 21b Practice Questions

Problem 1:

Let A denote the matrix $\begin{pmatrix} 1 & 4 \\ 1 & -2 \end{pmatrix}$

a) Find the eigenvalues and eigenvectors of A.

b) Solve the dynamical system $\vec{x}(m+1) = A\vec{x}(m)$ given that $\vec{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Thus, give $\vec{x}(m)$ for m = 1, 2, 3, … .

Problem 2:

Which of the following is the equation for the best fit as determined by the least squares method for a line through the four points $\begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ in the x-y plane? Please justify your work.

a) $y = \frac{14}{11}x + \frac{27}{22}$
b) $y = \frac{11}{13}x + \frac{27}{11}$
c) $y = \frac{13}{11}x + \frac{21}{11}$
d) $y = \frac{11}{14}x + \frac{21}{22}$
e) $y = \frac{13}{11}x + \frac{27}{11}$
f) $y = \frac{14}{11}x + \frac{21}{22}$

Problem 3:

The vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is the eigenvector with eigenvalue 3 of a symmetric 2 × 2 matrix with trace equal to 1. Write down the matrix.

Problem 4:

Circle **T** if the accompanying statement is true, and circle **F** if it is false. You need not justify your answers.

T F a) There are infinitely many 2×2 matrices with determinant equal to 1 and trace equal to 2.

T F b) All invertible matrices are diagonalisable.

T F c) A non-zero matrix with 2 columns and 4 rows must have 2-dimensional image.

T F d) SAS^{-1} is diagonal if $S = \frac{1}{4}\begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}$ and if A is a 2×2 matrix with eigenvectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Answers:

1. a) 2, -3 with eigenvectors $\vec{e}_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

b) $2^m \vec{e}_1 + 2(-3)^m \vec{e}_2$.

2. e)

3. $\begin{pmatrix} -1 & -2 \\ -2 & 2 \end{pmatrix}$

4. T, F, F, T.