

MATH 21b Midsemester Exam

F 2006

Practice questions

Problem 1:

Circle **T** if the given assertion is true, and circle **F** if it is false. There is no need to justify your answer.

- T F** a) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$. Then the equation $A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ has no solution.
- T F** b) Vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ in \mathbf{R}^4 are necessarily linearly independent if the following three conditions all hold: First, \vec{v}_1 is not in the span of $\{\vec{v}_2, \vec{v}_3\}$. Second, \vec{v}_2 is not in the span of $\{\vec{v}_3, \vec{v}_4\}$. Finally, \vec{v}_3 is not in the span of $\{\vec{v}_1, \vec{v}_2\}$.
- T F** c) If A is an $n \times n$ matrix and $A = AA$, then A must be either 0 or the identity matrix.
- T F** d) Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 6 & 4 \\ -1 & -4 & -3 \end{pmatrix}$. Then $\text{rref}(A) = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.
- T F** e) If a matrix has kernel $= \{0\}$, then it must be invertible.
- T F** f) The span of the rows of a matrix A must be the same as the span of the rows of $\text{rref}(A)$.
- T F** g) If A is a 2×2 matrix and AAA is the identity, then A must be the identity matrix.
- T F** h) If A and B are $n \times n$ matrices and AB is invertible, then so are A and B .
- T F** i) A linear transformation of \mathbf{R}^2 that sends $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ to $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ must send $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ to $\begin{pmatrix} 1 \\ 8 \end{pmatrix}$.

Problem 2: Let $\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Meanwhile, let $\vec{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$. This problem concerns the linear transformation, $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ that sends \vec{v}_1 to \vec{e}_1 and \vec{v}_2 to \vec{e}_2 .

a) Find the matrix of T with respect to the basis (\vec{e}_1, \vec{e}_2) .

b) Find a vector \vec{u} such that $T\vec{u} = \vec{v}_1$.

Problem 3: This problem concerns $\vec{v}_1 = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$, $\vec{v}_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$, $\vec{v}_4 = \begin{pmatrix} -5 \\ 0 \\ -1 \\ -2 \end{pmatrix}$, all

vectors in \mathbb{R}^4 .

- a) Write down a basis for the span of these four vectors.
 b) Circle the one vector in the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ of the four that follow.

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

- c) Let A denote the 4×4 matrix whose k 'th column is \vec{v}_k . Here, $k \in \{1, 2, 3, 4\}$. Give a basis for the kernel of A .
 d) Give a basis for the image of A .

Problem 4.

Circle T if the given assertion is true, and circle F if it is false. There is no need to justify your answer.

- T F a) Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Then $AB = BA$.
 T F b) Suppose \vec{u} is a non-zero vector in \mathbb{R}^n . The map of \mathbb{R}^n to itself that sends any given vector \vec{v} to $T(\vec{v}) = \vec{v} + \vec{u}$ is a linear transformation.
 T F c) If $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $S: \mathbb{R}^m \rightarrow \mathbb{R}^n$ are linear transformations such that the kernel of S has dimension greater than zero, then the kernel of ST must have dimension greater than zero.
 T F d) If the product of two matrices is 0, then one or the other must also be 0.
 T F e) If A is a matrix, then $\ker(A)$ must be the same subspace as $\ker(\text{rref}(A))$.
 T F f) If A is a matrix, then $\text{image}(A)$ must be the same as $\text{image}(\text{rref}(A))$.
 T F g) A linear transformation of \mathbb{R}^2 cannot send $\begin{pmatrix} 6 \\ -9 \end{pmatrix}$ to $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ to $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Problem 5.

Let A denote the matrix $\begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ 1 & 2 & 1 & -1 & 0 \\ 2 & 4 & -1 & 1 & 3 \\ 3 & 6 & -1 & -1 & 0 \end{pmatrix}$.

- Compute $\text{rref}(A)$.
- Give a basis for $\text{kernel}(A)$.
- Give a basis for $\text{image}(A)$.