Math 155: Designs and Groups

Homework Assignment #4 (22 February 2016):
Strongly Regular Graphs I

This problem set is due Monday, Feb. 29 [sic!] in class.

1. [Asserted at the bottom of page 34 of the textbook] Prove that a strongly regular graph with $\mu = 0$ must consist of r disjoint copies of the complete graph K_m for some $r, m > 1$.

2. If A is the incidence matrix of a graph G, what (in terms of A, I, J) is the incidence matrix of its complement $\overline{G}$? Use the incidence-matrix formulation of strong regularity to give an alternative proof of Prop. 2.7 (the strong regularity and parameters of the complement of a strongly regular graph).

3–6. Solve problems #3,4,6,5 on page 45 of the textbook. [For the first of these, see pages 41 and 42 for the relevant bounds; for the last, note that Ex. 2.21 is two paragraphs long... but you need not prove the identification with the Paley graph.[1]]

7. A *clique* in a graph G is a set of vertices of G any two of which are adjacent; a *co-clique* of G is a clique of the complementary graph $\overline{G}$, i.e. a set of vertices no two of which are adjacent in G. Prove that in a regular graph G of degree k on n vertices, any co-clique contains at most $-sn/(k-s)$ vertices, where s is the smallest eigenvalue of the adjacency matrix of G. [NB: G is <u>not</u> assumed *strongly* regular. Necessarily $s < 0$ (why?)]

8. Find a co-clique of size $-sn/(k-s)$ in each of the following graphs: the square lattice graph $L_2(m)$, the graph $r \cdot K_m$ of problem 1 above, and the Petersen graph. How many such maximal co-cliques are there in each case?

[1] Actually this is another typo in the textbook: the graph constructed in the first paragraph is a special case of Paley's construction, not the other way around.