

## Freshman Seminar 23j (Fall 2006–7): Chess and Mathematics

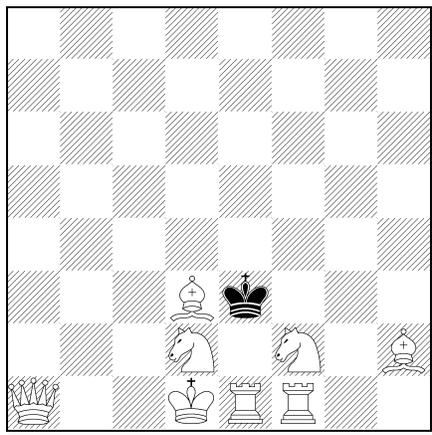
### Preliminary Puzzle(s)

Please answer as much of the following as you can; you'll probably find that the questions start out easy and get harder and (except for **1**) more open-ended. Instructions for e-mail are at the end.

**1** What is your favorite math and/or chess puzzle?

The remaining questions build on the following position:

**Diagram 1**



A remarkable checkmate

White has all 8 pieces<sup>1</sup> of the initial array; Black has only the King; all pieces are on the bottom three rows; and Black is checkmated. The position is *legal*: it can be reached from the initial array by game consisting entirely of legal moves, however contrived. None of this is all that remarkable; what is noteworthy about this checkmate position is that each of White's eight pieces is necessary for it to be a legal checkmate. For instance, without the Bh2,<sup>2</sup> the Black King could escape to f4.

**2** Explain why each of the remaining 7 White pieces is necessary. (Hint: What was White's last move?)

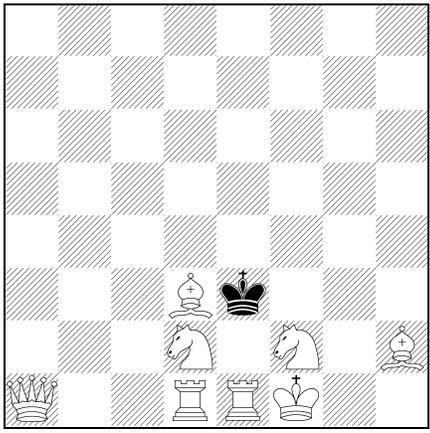
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<sup>1</sup>A pawn does not count as a "piece" (see the glossary at [http://www.math.harvard.edu/~elkies/FS23j.06/glossary\\_chess.html](http://www.math.harvard.edu/~elkies/FS23j.06/glossary_chess.html)).

<sup>2</sup>See [http://www.math.harvard.edu/~elkies/FS23j.06/alg\\_not.html](http://www.math.harvard.edu/~elkies/FS23j.06/alg_not.html)) if you are not familiar with the "algebraic notation" for chess.

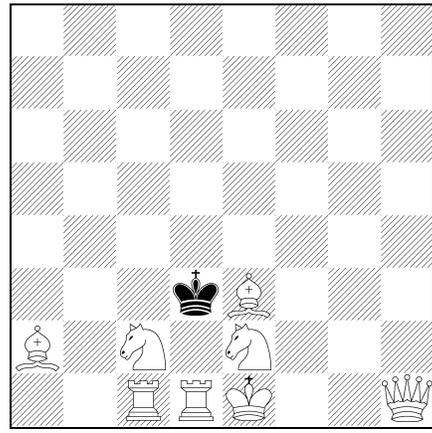
3 Diagram 1 allows some variations that retain all of the features noted in the first page. For instance, Kd1 and Rf1 can switch positions (see Diagram 1a), or the whole diagram can be reflected about a vertical axis (Diagram 1b).

Diagram 1a



Kd1 switches with Rf1

Diagram 1b

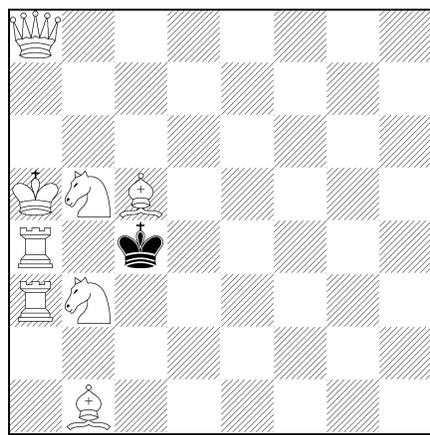


Reflected diagram

What is the total number of such versions?

4 Since no pawns appear in the position, we might expect that *rotating* the board should also preserve its features, with the pieces limited to the left or right three columns or the top three rows instead of the bottom three. For instance, a 90° clockwise rotation produces Diagram 1c:

Diagram 1c



Unusual checkmate, rotated

What is the total number of such versions? (Be careful with the top rows!)

**5** Can you find an entirely different way to set up a checkmate on the bottom three rows that has all of the features we observed for Diagram 1? If so, can you enumerate the variations of your setup (as you did earlier for question **3**)? Can you formulate — and perhaps answer — any further puzzles suggested by this?

**e-mail instructions.** You can e-mail your solutions, partial solutions, generalizations, conjectures, queries, and other comments to me at [elkies@math.harvard.edu](mailto:elkies@math.harvard.edu) . Please use *text only* if at all possible; do not gratuitously MIME your message, “attach” it, or duplicate it in HTML: I do not use a browser to read my e-mail, and mathematical writing is particularly hard to read when encrusted in MIME or HTML incantations. Whatever you do, **do not e-mail me a Micro\$oft Word (.doc) file**: I will not be able to read it. If you need to include a diagram, please either convert it to .pdf, or just write up your answer legibly by hand and then send or bring it to my mailbox or office in the Mathematics Department (both in the 3rd floor of the Science Center). Thank you.

Good luck!

—Noam D. Elkies