Problem Set # 8

M392C: K-theory

1. Fix a positive integer n. Let E denote the space of skew-Hermitian $n \times n$ matrices with operator norm ≤ 1. (The eigenvalues $i\lambda_1, \ldots, i\lambda_n$ satisfy $|\lambda_j| \leq 1$.) Consider the exponential map

$$\begin{aligned} p\colon E &\longrightarrow U(n) \\ A &\longmapsto \exp(\pi A) \end{aligned}$$

   (a) For each k between 0 and n prove that the restriction of p over the subspace of $U(n)$ consisting of unitary matrices with (-1)-eigenspace of dimension k is a fiber bundle. What is the fiber?

   (b) Show that p is a quasifibration.