Problem Set # 1

M392C: Bordism Old and New

Due: October 18, 2012

Complete at least 3 of the following problems, but best if you can do more!

1. Derive the signature formula for a closed oriented 8-manifold. You may use the result that $\Omega_8^{SO}\otimes\mathbb{Q}$ is 2-dimensional with basis the classes of $\mathbb{CP}^2 \times \mathbb{CP}^2$ and $\mathbb{CP}^4$.

2. Check the signature formula in the previous problem for the quaternionic projective plane $\mathbb{HP}^2$.

3. Suppose $V_1 \to M_1$ and $V_2 \to M_2$ are real vector bundles. Find a relationship among the Thom complexes $M_1^{V_1}$, $M_2^{V_2}$, and $(M_1 \times M_2)^{V_1 \times V_2}$.

4. Prove that $\mathbb{CP}^4$ does not embed in $\mathbb{A}^{11}$. (Hint: Consider Pontrjagin classes.)

5. Construct a 20-dimensional closed oriented manifold with signature 2012.

6. (a) Construct a double cover homomorphism $SU(2) \times U(1) \to U(2)$.

   (b) Compute the rational homotopy groups $\pi_i U(8) \otimes \mathbb{Q}$ for $i = 1, \ldots, 4$.

   (c) Compute as much of $H_\bullet\big(BU(8); \mathbb{Q}\big)$ as you can.