# QUIZ 2

Algebra — Math 122 — Fall 2002

1. F. If a finite group $G$ acts on a finite set $S$, then for any $s \in S$ we have $|G| = |\operatorname{Stab}(s)| \cdot |S|$. (You need $G$ to act transitively.)

2. T. The class equation for $D_5$ is $10 = 1+2+2+5$. (The conjugacy classes are the identity, 5 reflections, 2 rotations by $\pm 1/5$th of a revolution, and 2 rotations by $\pm 2/5$th of a revolution.)

3. F. If $H$ is a subgroup of $G$, then $N(H) = \{g \in G : gHg^{-1} = H\}$ is a normal subgroup of $G$. ($H$ is a normal subgroup of $N(H)$, but e.g. $N(H)$ is not normal in $G = S_5$ when $H = \langle (12) \rangle$.)

4. F. Any group of order 45 has at least 2 subgroups of order 5. (By the Sylow theorems, the number $s$ of such subgroups satisfies $s|9$ and $s = 1 \bmod 5$ so $s = 1$.)

5. F. The free group on 2 generators is a simple group. (The free group $F(a, b)$ maps, for example, to $\mathbb{Z}/2$ by sending $a$ and $b$ to 1. The kernel of this map is a nontrivial normal subgroup.)

6. F. For any $A \in GL_n(\mathbb{C})$, the matrix $T = AA^t$ is Hermitian. (For a complex matrix you need $AA^*$, not $A^t$.)

7. F. The bilinear form on $\mathbb{R}^2$ with matrix $A = \left(\begin{smallmatrix} 1 & 3 \\ 3 & 1 \end{smallmatrix}\right)$ is equivalent to dot product. (Since $\det(A) = 1 - 9 < 0$, $A$ is not positive-definite.)

8. F. There is a linear map of $\mathbb{R}^3$ to itself sending the locus $x^2 + y^2 + 2yz + z^2 = 1$ to the unit sphere. (The locus $x^2 + (y+z)^2 = 1$ is a cylinder.)