1. Let $\chi$ be a nontrivial Dirichlet character, $w$ a complex number, and $n$ a positive integer. Define

$$\varphi(n, w, \chi) := n^w \prod_{p \mid n} \frac{1 - \chi(p)}{p^w}.$$ 

Assume the Riemann hypothesis for the Riemann zeta function and Dirichlet $L$-functions.

1.1. Suppose $\Re(w) > 0$. Give an asymptotic for the partial sum $\sum_{n<X} \varphi(n, w, \chi)$ with a power-saving error term, i.e. prove a statement of the form

$$\sum_{n<X} \varphi(n, w, \chi) = cX^\alpha + O(X^\beta)$$

for some $\alpha, \beta, c$ with $\Re(\alpha) > \Re(\beta)$.

1.2.* Repeat problem 1.1 with the assumption that $-\frac{1}{2} < \Re(w) < 0$. For a lower bound on $L$-functions in the critical strip, you can use Iwaniec–Kowalski theorem 5.19.