Math 131 - Problem Set 6
Due Tuesday, Oct 16

From Munkres: 28.7abc, 29.1, 29.3, 29.6, 29.8

1. Let X be a compact, connected, Hausdorff topological space, and let $Y = X \cup \{\infty\}$ be its one-point compactification. Show that Y is not connected. What are the connected components of Y ?

2. A subset $X \subset Y$ of a topological space Y is said to be *locally closed* if for any point $p \in X$, there is an open subset $U \subset Y$ such that $p \in U$ and $X \cap U$ is closed in U.

   (a) Are the following subsets of $Y = \mathbb{R}^2$ locally closed?

      i. $X = \{(x, 0) \mid x \neq 0\}$
      ii. $X = \{(1/n, 0) \mid n \in \mathbb{Z}_+\}$
      iii. $X = \{(x, y) \mid x \neq 0\} \cup \{(0, 0)\}$

   (b) Show that a locally closed subset $X \subset Y$ is the intersection of a closed subset of Y and an open subset of Y.