Math 131 - Problem Set 3
Due Tuesday, Sept 25

From Munkres: 17.11, 17.13, 17.20, 19.6, 19.7, 20.4

1. The *Zariski topology* on $\mathbb{R}^2$ is defined to be the topology with basis

$$U_f = \{(x, y) \in \mathbb{R}^2 \mid f(x, y) \neq 0\}$$

   where f ranges over all polynomials $f \in \mathbb{R}[x, y]$.

   (a) Show that the subsets $U_f \subset \mathbb{R}^2$ are in fact a basis.

   (b) Show that this topology is not Hausdorff.

   (c) If $\mathbb{R} \subset \mathbb{R}^2$ is any line, show that the subspace topology on $\mathbb{R}$ induced by the Zariski topology is the finite complement topology.

   (d) Let

$$\Phi = \{(x, y) \in \mathbb{R}^2 \mid y = \sin x\}.$$

   What is the closure of Φ in the Zariski topology?