

## Math 131 - Problem Set 2

### Due Tuesday, Sept 18

From Munkres: 13.8, 16.10, 18.3, 18.6

1. Let  $X$  be a topological space and  $Y \subset X$  any subset. Show that the subspace topology induced on  $Y$  is the coarsest topology on  $Y$  such that the inclusion map  $Y \hookrightarrow X$  is continuous.
2. Let  $(X, d)$  be a metric space and  $A \subset X$ . Show that the topology on  $A$  induced by the metric  $d|_{A \times A}$  is the subspace topology.
3. Let  $X$  and  $Y$  be topological spaces, and let  $p_1 : X \times Y \rightarrow X$  and  $p_2 : X \times Y \rightarrow Y$  be the two projection maps. Show that the product topology on  $X \times Y$  is the coarsest topology such that  $p_1$  and  $p_2$  are continuous.
4. Consider  $\mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$  given the order topology (with the dictionary order). In each case, determine whether or not the given subset is open. You don't need rigorous proofs here, just convincing explanations. These are mostly just fun examples to think about.
  - (a)  $U = [1, 2] \times (1, 2)$
  - (b)  $U = (1, 2) \times (1, 2)$
  - (c)  $U = (1, 2) \times [1, 2]$
  - (d)  $U = [0, 1) \times [0, \infty)$