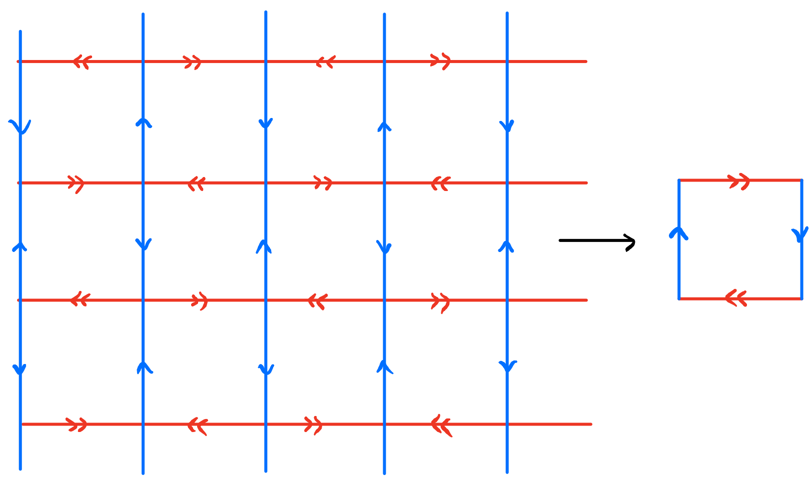
Math 131 - Problem Set 11
Due Thursday, December 6

From Munkres: 79.3, 71.2, 71.3

1. Let G and H be free groups on m and n generators respectively. Show that $G \cong H$ if and only if $m = n$. (You may prove this using group theory or topology.)

2. Show that $\mathbb{RP}^2$ cannot be covered by a torus or by $\mathbb{R}^2$.

3. In light of problem 2., why doesn't the following diagram describe a covering of $\mathbb{RP}^2$ by $\mathbb{R}^2$?



4. Compute the fundamental group of the complement of 2 points in the following spaces. (You don't need to explicitly write the deformation retractions you use. Pictures are acceptable.)

   (a) $\mathbb{R}^2$

   (b) S^2

   (c) A torus

5. Show that the free group on two generators has an infinitely generated subgroup. (Hint: Consider coverings of the figure-eight space.)