Math 131 — Final Exam

December 7-10 (or 10-13), 2018

Name: ____________________

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1. This exam has 8 pages including this cover. There are 5 problems.
2. In your solutions, you may refer to any of the theorems proved in class or on the homework.
3. **You may not use any external sources on this exam other than the course materials (class notes, problem sets, and Munkres).** You may not use any other textbooks or external websites and you may not discuss the problems with anybody.
4. This exam is due in my office (SC 507) by 2 PM on December 10 (December 13 for the late exam).
5. Make sure you read and sign the last page before turning in the exam.

Problem	Points	Score
1	5	
2	8	
3	8	
4	7	
5	7	
Total	35	

1. [5 points] Let $X = B^2 \times S^1$ be the *solid torus*, with boundary $Y = S^1 \times S^1$, and inclusion map $i : Y \to X$. Show that there is no retraction $r : X \to Y$.

2. [8 points] Compute the fundamental groups of the following spaces by giving a presentation in terms of loops on the space.

a. [4 points] $A =$ The Möbius band minus a point.

b. [4 points] $B =$ the Klein bottle.

3. [8 points] Determine whether the following statements are true or false. If true, give a **brief** justification. If false, give an explanation or counterexample, if appropriate.

a. [2 points] If $f : [0,1] \to X$ is a loop at x_0 and f is nulhomotopic, then $[f]$ is the identity in the group $\pi_1(X, x_0)$.

b. [2 points] Every covering map $p : S^2 \to S^2$ is a homeomorphism.

c. [2 points] The fundamental group of $\mathbb{RP}^2 \vee \mathbb{RP}^2$ is infinite.

d. [2 points] If $f : [0,1] \to X$ is a loop, and $g : [0,1] \to S^1$ is defined $g(x) = (\cos(2\pi x), \sin(2\pi x))$, then there is a function $h : S^1 \to X$ such that $h \circ g = f$.

4. [7 points] Let X be a topological space with basepoint x_0. Let f be a loop at x_0. Let $h : [0, 1] \to [0, 1]$ be a homeomorphism. Show that $[f \circ h] = [f]$ or $[f]^{-1}$ in $\pi_1(X, x_0)$.

5. [7 points] Show that there is no continuous injective function $f : S^2 \to S^1 \times S^1$.

Before handing in your exam, please read the following and sign below.

While completing this exam, I have not consulted any external sources other than class notes, the textbook (Munkres), and problem sets. I have not discussed the problems or solutions of this exam with anyone.

Sign here: ____