**Math 101 - Problem Set 8**
**Due Tuesday, November 7**

1. Consider the following groups of order 8: $D_8, Q_8, \mathbb{Z}_8, \mathbb{Z}_4 \times \mathbb{Z}_2$. For each pair of groups, either show that the two groups are isomorphic, or explain why they're not.

2. Determine whether each of the following functions is a homomorphism or not.

   (a) $\phi : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$ under multiplication, defined $\phi(x) =$ the absolute value of $x$.

   (b) $\phi : \mathbb{R} \to \mathbb{Z}$ under addition, defined $\phi(x) =$ the greatest integer less than or equal to $x$ (For example, $\phi(3.7) = 3$).

   (c) $\phi : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ under addition, defined $\phi(x, y) = x - y$.

   (d) $\phi : D_8 \to \mathbb{Z}_4$ defined $\phi(r^i s^j) = i \pmod 4$.

3. For each of the functions in the previous problem that are in fact homomorphisms, describe the kernel and the image.

4. Let $G$ be a group. Show that the function $f : G \to G$ defined $f(x) = x^{-1}$ is a homomorphism if and only if $G$ is abelian.

5. Show that $\mathbb{Z} \times \mathbb{Z}$ (under addition) is not cyclic.

6. Show that every subgroup of a cyclic group is cyclic.

7. Using the statement you proved in the previous problem, list each subgroup of $\mathbb{Z}_8$. How many distinct subgroups are there? How many elements of $\mathbb{Z}_8$ generate the entire group?