Math 101 - Problem Set 7
Due Tuesday, October 31

1. Let $x \in D_{2n}$ be an element that is not a power of r. Use the properties of the dihedral group that we proved in class to show the following.

   (a) x has order 2.

   (b) $rx = xr^{-1}$.

2. Let G be the group of rigid motions of a cube. (i.e. label the 8 vertices, and rotate the cube in any fashion as long as it sits in the same orientation, just as we did with the square in class.) Calculate $|G|$. Make sure you explain your answer.

3. For each of the following subsets H of the given group G, either prove that H is a subgroup of G or show it's not a subgroup.

   (a) $G = \langle \mathbb{R} - \{0\}, \cdot \rangle$ and $H = \{x \in \mathbb{R} - \{0\} \mid x^2 \in \mathbb{Q}\}$.

   (b) $G = \langle \mathbb{R}, + \rangle$ and $H = \{x \in \mathbb{R} \mid x^2 \in \mathbb{Q}\}$.

   (c) $G = D_{2n}$ and $H = \{r^i s \mid i \in \{0, 1, \ldots, n-1\}\} \cup \{e\}$.

   (d) $G = \langle \{f : \mathbb{R} \to \mathbb{R}\}, + \rangle$ and $H = \{f \in G \mid f(1) = 0\}$.

4. Let G be a finite group such that $|G| = n > 2$. Show that there is no subgroup H such that $|H| = n - 1$.

5. Let G be an abelian group. Let H be the subset of G consisting of all elements of finite order. Show that $H \leq G$. This is called the ***torsion subgroup*** of G.

6. Let $n \in \mathbb{Z}_+$, and consider the group $G = \mathbb{Z} \times \mathbb{Z}_n$. Find the torsion subgroup of G.

7. Let H and K be subgroups of G. Show that $H \cap K$ is a subgroup of G.

8. Let A be an abelian group and fix $n \in \mathbb{Z}$. Prove that the given sets are subgroups of A.

   (a) $\{a^n \mid a \in A\}$

   (b) $\{a \in A \mid a^n = e\}$

9. Show that if $n \geq 3$, then $\{x \in D_{2n} \mid x^2 = e\}$ is not a subgroup of D_{2n}. (Thus, 8.(b) is not true if the abelian assumption is removed.)