Math 101 - Problem Set 5
Due Tuesday, October 17

1. Let A and B be finite sets. Show that the set of all functions $f : A \to B$ is finite.

2. Let A be a set of cardinality n, for some $n \in \mathbb{Z}_+$. Suppose A has a total ordering $\leq$.

   (a) Show that A has a largest element and a smallest element.

   (b) Show that A has the same order type as $\{1, 2, \ldots, n\}$ equipped with the standard ordering.

3. Show that every nonempty subset $A \subset \mathbb{Z}_+$ has a smallest element.
   (*Hint: first show that every subset of* $\{1, \ldots, n\}$ *has a smallest element.*)

4. For each of the following subset of $\mathbb{R}^\omega$, either express it as the cartesian product of subsets of $\mathbb{R}$, or explain why that is not possible. Let $\mathbf{x}$ denote the ω-tuple $(x_i)_{i \in \mathbb{Z}_+}$.

   (a) $\{\mathbf{x} \mid x_i \text{ is an integer for all } i\}$

   (b) $\{\mathbf{x} \mid x_i \geq i \text{ for all } i\}$

   (c) $\{\mathbf{x} \mid x_i \text{ is an integer for all } i \geq 100\}$

   (d) $\{\mathbf{x} \mid x_1 = x_2\}$

5. Show that $\{0, 1\}^\omega$ has the same cardinality as $\mathscr{P}(\mathbb{Z}_+)$ by constructing a bijection between them.

6. Let $n \in \mathbb{Z}_+$ and let $A_1, A_2, \ldots, A_n$ be nonempty countable sets. Show that the cartesian product $A_1 \times A_2 \times \cdots \times A_n$ is countable.

7. Show that the following sets are countable.

   (a) The set A_n of all n-element subsets of $\mathbb{Z}_+$ (for $n \in \mathbb{Z}_+$).

   (b) The set B of all finite subsets of $\mathbb{Z}_+$.

8. Show that the set of functions $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ is uncountable.