

Math 101 — Midterm Exam 2

November 9, 2017

Name: _____

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1. **Do not open this exam until you are told to do so.**
 2. This exam has 6 pages including this cover. There are 5 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
 3. In your solutions, you may refer to any of the theorems proved in class or on the homework.
 4. You may use no aids (e.g., calculators or notecards) on this exam.
 5. **Turn off all cell phones.**
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Problem	Points	Score
1	8	
2	7	
3	8	
4	10	
5	7	
Total	40	

2. [7 points] Let G be an abelian group. Show that for all $a, b \in G$, $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}_+$.

3. [8 points] Let A , B , and C be groups, and let $f : A \rightarrow B$ and $g : B \rightarrow C$ be homomorphisms. Define $\varphi = g \circ f$.

a. [2 points] Show that $\varphi : A \rightarrow C$ is a homomorphism.

b. [6 points] Assume g is injective. Show $\text{Ker}(\varphi) = \text{Ker}(f)$.

5. [7 points] Let G be a group and H and K subgroups of G . Show that if $H \cup K$ is a subgroup of G then $H \subseteq K$ or $K \subseteq H$.

[Hint: Assume neither set contains the other. Then we can take $x \in K - H$ and $y \in H - K$.]