Math 101 — Midterm Exam 1

October 5, 2017

Name: ______________________________

1. **Do not open this exam until you are told to do so.**
2. This exam has 6 pages including this cover. There are 5 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. In your solutions, you may refer to any of the theorems proved in class or on the homework.
4. You may use no aids (e.g., calculators or notecards) on this exam.
5. **Turn off all cell phones**.

Problem	Points	Score
1	8	
2	9	
3	7	
4	8	
5	8	
Total	40	

1. [8 points] Short answer. For each part of this problem, you do not need to justify your answer.

a. [2 points] Write the negation of the following sentence: "If $x, y \in \mathbb{Z}$ such that $y < x$, then there is some $z \in \mathbb{Q}$ such that $y < z < x$."

b. [2 points] Let A and B be sets and $f : A \to B$ a function. Define the equivalence relation $\sim$ on A by $a_1 \sim a_2$ if and only if $f(a_1) = f(a_2)$. Let $g : B \to A/\sim$ be the function we defined in class. Complete the following sentence: "g is well-defined if and only if . . ."

c. [2 points] Give an example of a relation that is reflexive but is neither symmetric nor transitive.

d. [2 points] List the elements of $\mathscr{P}(\mathscr{P}(\mathscr{P}(\emptyset)))$, where $\emptyset$ is the empty set.

2. [9 points] Let A, B, and C be sets and $f : A \to B$ and $g : B \to C$.

a. [7 points] Show that if $g \circ f$ is injective and f is surjective, then g is injective.

b. [2 points] Give an example of functions $g : B \to C$ and $f : A \to B$ where $g \circ f$ is injective, but g is not injective. Make sure to specify what A, B, and C are in your example. You do not need to justify your answer.

3. [7 points] Let A and B be ordered sets that have the same order type. Show that if $b \in B$ has an immediate predecessor, there is some element of A that also has an immediate predecessor.

4. [8 points] Show that if an ordered set X has the greatest lower bound property, then it has the least upper bound property. (This is the converse of a statement we proved in class.)

5. [8 points] Determine whether the following statements are true or false, and briefly justify your answer (or give a counterexample, if applicable).

a. [2 points] The relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ defined $(x, y) \sim (z, w)$ if and only if $z = ax$ and $w = ay$ for some $a \in \mathbb{R}$ is an equivalence relation.

b. [2 points] If $f : A \to B$ is a function, and $A_0 \subseteq A$, then $f(A - A_0) = f(A) - f(A_0)$.

c. [2 points] Let A be a set and $\mathcal{C}$ a partition of A. Then $\{\emptyset\} \notin \mathcal{C}$.

d. [2 points] The set $\mathbb{Z}_+ \times \mathbb{Z}_+$ with the dictionary order has the same order type as $\mathbb{Z}_+ \times \mathbb{Z}_-$ with the dictionary order, where $\mathbb{Z}_- = \{x \in \mathbb{Z} \mid x < 0\}$.