**Problem 1.** Fix a point $p \in S$ and let $C$ be a curve through $p$. Show that the geodesic curvature $\kappa_g(p)$ of $C$ at $p$ is equal to the curvature $\kappa(p)$ at $p$ of the normal projection of $C$ onto the tangent plane $T_pS$.

**Problem 2.** Suppose $S$ is a surface with metric $g$ and suppose that we have isotherman coordinates $(u^1, u^2)$, where the metric takes the form

$$(g_{ij}) = \lambda(u^1, u^2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where $\lambda > 0$ is a non-negative smooth function.

1. Write down the geodesic equations in this chart.
2. Consider the special case $S = \{u^2 > 0\} \subset \mathbb{R}^2$ and $\lambda = u^2$. Note here this metric may not necessarily arise from a parametrization of a surface in $\mathbb{R}^3$ but we can still compute with it as an abstract metric. Compute the geodesic $\gamma(t) = (u^1(t), u^2(t))$ with initial condition $\gamma(0) = (a, b)$ and $\dot{\gamma}(0) = (0, \frac{1}{\sqrt{b}})$, $b > 0$.

The **Poincaré upper half plane** is $\{(x, y) \in \mathbb{R}^2 \mid y > 0\}$ equipped with the abstract metric

$$g = \frac{1}{y^2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Problem 3.** Show that the geodesic equations for the upper half plane can be written as

$$\frac{d}{ds} \frac{x'}{y^2} = 0 \quad \frac{d^2 y}{ds^2} = \frac{1}{y}(x'^2 - (y')^2)$$

where $\gamma(s) = (x(s), y(s))$ is an arc-length parametrized curve.

**Problem 4.** Show that the geodesics on the upper half plane are vertical lines as well as semi-circles centered at a point on the $x$-axis. Hint: what is the equation of a circle with center $(b, 0)$ and radius $r$?

**Problem 5.** Calculate the Gaussian curvature $K$ of the upper half plane.