July 20

Problem 1.

Show that the following are subtractive systems.

(a) $(\mathcal{P}(X), \cup, \setminus\ )$ for any set X.

(b) $(L(\mathcal{H}), \widehat{+}, \setminus\ )$ for any Hilbert space $\mathcal{H}$.

You can use the following properties of subspaces A, B of a Hilbert space:

(i) $A^{\perp\perp} = \overline{A}$.

(ii) $(A+B)^{\perp} = A^{\perp} \cap B^{\perp}$.

(iii) $(A \cap B)^{\perp} = \overline{A^{\perp} + B^{\perp}}$.

(iv) If $A \perp B$, then $\overline{A+B} = \overline{A} + \overline{B}$.

Problem 2.

Set $[n] = \{1, ..., n\}$. Let $\mathcal{H}$ be an m-dimensional Hilbert space with $m \geq n$. Describe the d-maps from $\mathcal{P}([n])$ to $L(\mathcal{H})$.