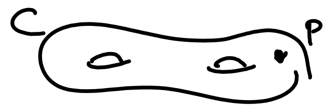
S. Gukov – 25/1/08 – Talk III

Recall: on $\mathcal{M}_H(G, C; p, \alpha, \beta, \gamma, \eta)$

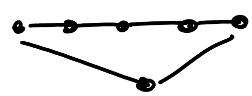


Model	Complex moduli	Kähler moduli
I	$\beta + i\gamma$	$\alpha + i\eta$
J	$\gamma + i\alpha$	$\beta + i\eta$
K	$\alpha + i\beta$	$\gamma + i\eta$

<u>Claim:</u>

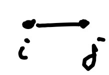
$$B_{aff} \circlearrowright D^b(\mathcal{M}_H(\ldots))$$
$$H_{aff} \circlearrowright K^{\mathbb{C}^*}(\mathcal{M}_H(\ldots))$$
$$W_{aff} \circlearrowright K(\mathcal{M}_H(\ldots))$$

<u>where:</u>
- W_{aff} = affine Weyl group $= \Lambda \rtimes W$
  Weyl gr. assoc^d to affine Dynkin diagram:
  e.g. for GL_n,



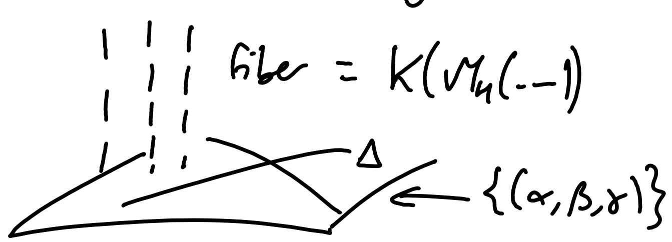
$T_i^2 = 1 \quad \forall i$

braid relations $\left\{ \begin{array}{ll} T_i T_j = T_j T_i & \text{no edge} \\ T_i T_j T_i = T_j T_i T_j & \end{array} \right.$



- $H_{aff} = (T_i - q)(T_i + 1) = 0$ & braid relations
- B_{aff} = affine braid group (keep only braid relations)
  eg. for GL_n, get usual braids on a <u>cylinder</u>.

- Realize W_{aff}-action on $K(\mathcal{M}_H)$ as <u>geometric monodromy</u>
  in moduli of parameters $(\alpha, \beta, \gamma) \in (\mathbb{T} \times \mathfrak{t} \times \mathfrak{t}) / W_{aff}$



$K(\mathcal{M}_H(\dots))$ jumps when $\mathcal{M}_H$ becomes singular

$\Longleftrightarrow$ when (α,β,γ) is fixed by an elt of W_{aff}.

Let Δ = discriminant locus $= \{(\alpha,\beta,\gamma) \,/\, \text{Stab} \neq \{1\} \subset W_{aff}\}$

has codim. ≥ 3 since each of α, β, γ must be stabilized.

Then monodromy action:

$$\pi_1\left((\mathfrak{t}^3 \setminus \Delta)/W_{aff}\right) \longrightarrow \text{Aut}\, K(\mathcal{M}_H)$$
$$\wr\!\wr \quad \text{codim}\,\Delta \geq 3$$
$$\pi_1(\mathfrak{t}^3/W_{aff})$$
$$\wr\!\wr$$
$$W_{aff}$$

✓

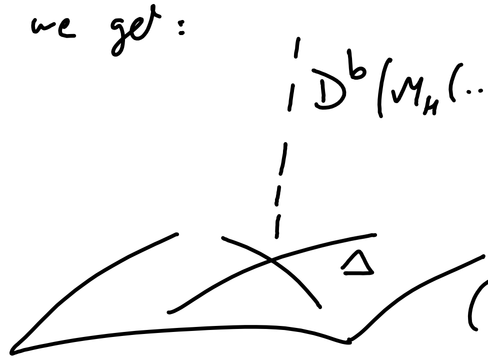
- Now look at $D^b(\mathcal{M}_H(\dots)_J)$

  $\rightsquigarrow$ here need to fix the $\mathbb{C}$ structure to avoid jumps

  $\rightarrow$ fix the complex moduli $\underline{\gamma + i\alpha = 0}$.

Now we get:

$D^b(\mathcal{M}_H(\dots)_J)$



$(\beta,\eta)^{\text{regular}} \in (\mathbb{T}_{\mathbb{C}} - \Delta)/W$

codim. 2

Classically D^b doesn't depend on the Kähler moduli $\beta + i\eta$, but at level of quantum corrections it does.

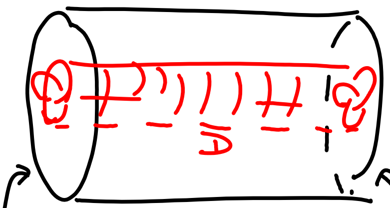
Easiest way to see this = $(\beta,\eta)^{reg}$ define a <u>stability cond.</u> and we vary the stability cond. on D^b.

Get induced action $B_{aff} = \pi_1\left((\mathbb{T}_{\mathbb{C}} - \Delta)/W\right)$ on D^b.

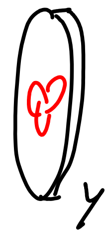
<u>Ex:</u> $G = SU(2)$, $C = D^* \rightsquigarrow \mathcal{M}_H \simeq T^*\mathbb{C}P^1$

DIM! REDUCTION :

- 4D TQFT on $M = Y \times \underset{[0,1]}{\overset{}{I}}$ $\longleftrightarrow$ 3D TQFT on Y



$\cong$



Y

Chern-Simons theory with group G.

$\mathcal{B}_{c.c.}$ $\qquad$ $\mathcal{B}'$: $\{\phi = 0\}$

$F^+_{A|Y} = 0$

surface operator supported on $D = K \times I$ $\rightsquigarrow$ line operator in CS theory on Y:

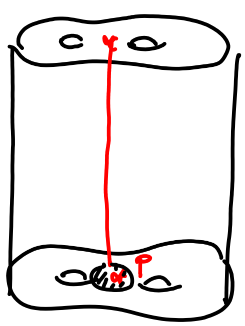
$$\begin{matrix} D = K \times I \\ \cap \quad\; \cap \\ M = Y \times I \end{matrix}$$

$W_R(K)$ - Wilson operators

R = rep^n of G

- If $Y = \mathbb{R} \times C$:

time



C

$\rightsquigarrow$ $\mathcal{H}$ Hilbert space.

If we look at a small nbd of p, replace C by D^* $\longrightarrow$ $\mathcal{H}$ = rep^n space of R.

So... looking at 4D theory on $\Sigma \times C$, reduces to σ-model $\mathrm{Maps}(\Sigma, \mathcal{M}_H(G,C))$

As in above picture, associate:

$\mathcal{H} = \mathrm{Hom}(\mathcal{B}_{cc}, \mathcal{B}')$ $\qquad$ branes on $\mathcal{M}_H(G,C)$.

- Take $\underline{C = D^*}$: then $\mathcal{M}_H \simeq T^*(G/_{T})$

(e.g. for $G = SU(2)$, $\mathcal{M}_H = T^*\mathbb{CP}^1$)

- $\mathcal{B}' = \{\phi = 0\}$ brane supported on $G/_{T} \subset T^*(G/_{T}) \simeq M_H$.
(A-brane)

- $\mathcal{B}_{c.c.}$ = canonical coisotropic brane on $\mathcal{M}_H$ [Kapustin-Orlov]
satisfies $(F \cdot \omega^{-1})^2 = -1$ $[\omega_I] = \alpha$
$[\omega_J] = \beta$
A-model brane in $\omega = \omega_K$ $[\omega_K] = \gamma$
curvature $F = \omega_J$

☆ $\mathcal{B}'$, $\mathcal{B}_{c.c.}$ are branes of type (A, B, A) wrt (I, J, K).
Using B-model (J), $\mathcal{H} = \mathrm{Hom}(\mathcal{B}_{cc}, \mathcal{B}')$
= holom. sections of a line bundle on $G/_{T}$

(Borel-Weil-Bott theory: construct rep^{ns} of G by looking a sections of a bundle over a mfld with G-action).

- In general, A-model for $\omega = \omega_K$ ("A_K-model"):

$$\left\{ \begin{matrix} K_{\mathbb{R}}\text{-invariant} \\ A_K\text{-branes} \end{matrix} \right\}_{\lambda = \gamma + i\eta} \longleftrightarrow \left\{ \begin{matrix} \text{Harish-Chandra} \\ \text{modules} \end{matrix} \right\}_{\lambda}$$

$$\mathcal{B}' \longmapsto \mathrm{Hom}(\mathcal{B}', \mathcal{B}_{cc}).$$

<u>Example:</u> $G = SU(2)$, $G_{\mathbb{C}} = SL(2, \mathbb{C})$, $G_{\mathbb{R}} = SL(2, \mathbb{R})$
$K_{\mathbb{R}} = SO(2)$

$$\mathcal{M}_H = \mathbb{H}^2 /\!/\!/ U(1) \simeq T^* S^2$$

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