21/1/08 Discussion session

1. **D. Orlov**: m.s. and strange Arnold duality:

$\mathbb{A}^n \xrightarrow{W} \mathbb{C}$ LG-model s.t. 1) $W = \sum_{i=1}^n u_i$, $u_i \in \mathbb{C}[x_1 \dots x_n]$ monomials

\# terms = \# variables

2) W has an isolated sing. at 0

3) u_i is never of the form $x_j x_k$, $j \neq k$.

$\rightarrow$ then each $u_i = x_i^{p_i} x_{\sigma(i)}$, $\sigma: \{1 \dots n\} \to \{1 \dots n\}$

s.t. $\forall j$, $\sigma^{-1}(j)$ = one of $\begin{cases} \phi \\ \{i\} \\ \{i,j\} \end{cases}$

$A = \mathbb{C}[x_1 \dots x_n]/W$

max. grading abelian group $L = \overbrace{\langle d_1, \dots, d_n, a \rangle}^{\mathbb{Z}^{n+1}} \Big/ \{p_i d_i + d_{\sigma(i)} = a\}$

$L \cong \mathbb{Z} \oplus$ Torsion

$D^L_{sg}(A) = D^b(\mathrm{mod}^L\text{-}A) / \mathrm{Perf}^L(A)$ = B-branes

L-graded der. cat. of singularities

Conj: $D^L_{sg}(A)$ has a full exc. collection

Mirror: $\mathbb{A}^n \ni (\tilde{x}_1 \dots \tilde{x}_n)$, $\mathbb{A}^n \xrightarrow{\tilde{W}} \mathbb{C}$, $\tilde{W} = \sum_{i=1}^n \tilde{u}_i + \lambda \prod_1^n \tilde{x}_i$, $\lambda \in \mathbb{C}$ small

where $\tilde{u}_i = \tilde{x}_i^{p_i} \prod_{j \in \sigma^{-1}(i)} \tilde{x}_j$ i.e. $\tilde{u}_i = \begin{cases} \tilde{x}_i^{p_i} & \text{if } \sigma^{-1}(i) = \phi \\ \tilde{x}_i^{p_i} \tilde{x}_j & \{j\} \\ \tilde{x}_i^{p_i+1} \tilde{x}_j & \{i,j\} \end{cases}$

Conj: $D^L_{sg}(A) \cong DF(\tilde{W})$.

Ex: case $\sigma = id$ gives: $W = \sum x_i^{p_i+1}$ with $\mathbb{Z} \oplus T$ grading

(or equivalently, $\left(\sum x_i^{p_i+1}\right)/T$)

since wal T-equivt part .....

$\updownarrow$

$$\tilde{W} = \sum \tilde{x}_i^{p_i+1} + \lambda \prod \tilde{x}_i$$

can prove by setting $\lambda = 0$ & splitting as $\prod_{i=1}^{n} (\mathbb{C}, x^{p_i})$.

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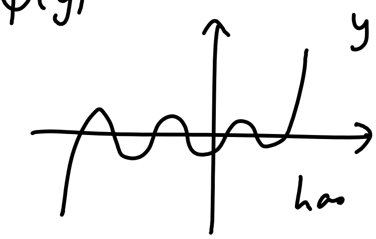
② Seidel: real Morsification (A'Campo) gives very compact exc. coll^ns for Fukaya cats. of singularities in 2 variables:

e.g. for A_n, $f(x,y) = x^2 + y^{n+1} + \psi(y)$

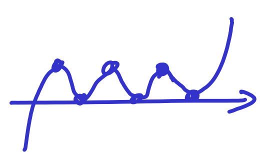
where $\deg \psi \leq n$ and ψ real



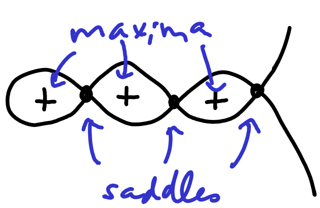
has max. # real roots.

$f_{\mathbb{R}}$ has maxima, minima, & saddles;
make it so that all saddle points have $f(\text{saddle}) = 0$

prevents heteroclinic trajectories saddle $\to$ saddle.

ie. 

Then $\text{Morse}(f_{\mathbb{R}}) \simeq \text{Fuk}(f_{\mathbb{C}})$

so for A_n: 

use this to analyze the above?

$\bullet \to \bullet \leftarrow \bullet \to \bullet \leftarrow \bullet \to \bullet$ or equivly,