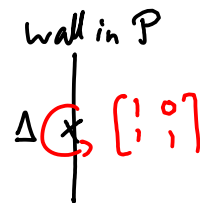


Ingredients:

- $(B, P, \varphi)$  "polarized" tropical mfd"
  - polyhedral decom<sup>n</sup> of  $B$
  - piecewise affine concave function
- $\mathbb{Z}$ -affine str. on  $B - \Delta$ , codim  $\Delta = 2$ .
- positivity (necessary), "simple"

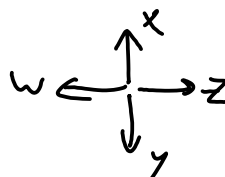
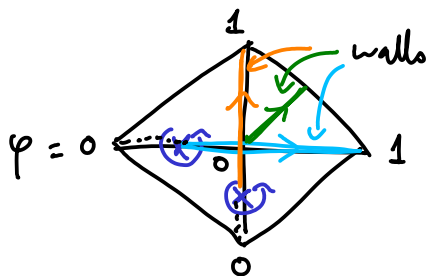


- standard (thickened) rings  $R_{w \rightarrow \tau, \sigma}^k$   
 $w \subset \tau \subset \sigma \in P_{max}$

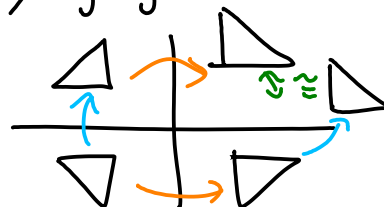
[  $\tau \leftrightarrow$  toric stratum of  $X_0$   
 $w \leftrightarrow$  open subset ]

- standard hom's  $R_{w \rightarrow \tau, \sigma}^k \rightarrow R_{w \rightarrow \tau', \sigma}^k$  for  $\tau \subset \tau'$   
 $R_{w \rightarrow \tau, \sigma}^k \hookrightarrow R_{w' \rightarrow \tau, \sigma}^k$  for  $w \subset w'$  localization

Example:



get  $X_k$  by gluing pieces w/ wall-crossings:



Correction  $\rightarrow$   $xy - (1+w)t$

## Algorithm:

• starting data  $\leftrightarrow f_p, p \in \mathcal{P}^{[n-1]}$  (log. str.)

• inductively insert walls (scattering procedure)  
and change  $f_p$ 's to higher order  $\rightsquigarrow \tilde{f}_p$ .

NB: outside of toric case like this,  
we'll get only many walls

• normalization process.

$\text{Log}(\tilde{f}_p)$  contains no pure  $t$ -terms  $\rightsquigarrow t$  canonically

$\rightsquigarrow$  "structure" to order  $k$ ,  $\mathcal{Y}_k = \{\text{walls}, \tilde{f}_p\}$