Math 53 Homework 12

Due **Wednesday 4/20/16** in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

- **Work:** Problem 5 of HW 11 (postponed to this assignment)

Monday 4/11: Surface area

- **Read:** section 16.6.
- **Work:** 16.6: (3), 13, (18), 23, 24[1], (25), (32), (39), 44[1], 45, (47).[2]

Wednesday 4/13: Surface integrals and flux

- **Read:** section 16.7.
- **Work:** 16.7: 16[3], (17), 18[3], (20), 23, 24[3], 26[3], (27), 29, (31), (32).[4]
  Problem 1 below.

Friday 4/15: The divergence theorem

- **Read:** section 16.9.
- **Work:** 16.9: (1), 2[5] , 3[5], (4), (5), 7, 11[5], (13).

Problem 1. (Surface area on the sphere.)

a) What percentage, rounded to the nearest percent, of the Earth's surface is north of Berkeley? The latitude here is 38°. (Latitude is related to the spherical angle ϕ by the formula: $\alpha = 90° - \phi$)

b) Find the average latitude of all points in the Southern Hemisphere.

Optional: Identify a city whose latitude is within one degree of the average.

Problem 5 of HW 11. (due with this assignment)

Find the flux of the vector field $\vec{F} = \dfrac{x\hat{\imath} + y\hat{\jmath}}{x^2 + y^2}$ outwards through the circle centered at $(1, 0)$ of radius $a \neq 1$. Consider the cases $a > 1$ and $a < 1$ separately, and use Green's theorem (carefully!). Explain your answers with diagrams.

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[1] **6th/7th ed:** do the 8th ed problems: **# 24:** the part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$. **# 44:** the part of the surface $z = 4 - 2x^2 + y$ that lies above the triangle with vertices (0,0), (1,0), and (1,1).

[2] **6th ed:** 16.6: (3), 13, (18), 23, 24[1], (25), (32), (37), 42[1], 41, (45). **7th:** same as 8th.

[3] **6th/7th ed:** do the 8th ed problems: **# 16:** $\iint_S y^2\, dS$, S is the part of the sphere $x^2+y^2+z^2 = 1$ that lies above the cone $z = \sqrt{x^2 + y^2}$. **# 18:** $\iint_S (x + y + z)\, dS$, S is the part of the half-cylinder $x^2+z^2 = 1$, $z \geq 0$, that lies between the planes $y = 0$ and $y = 2$. **# 24:** $\vec{F}(x, y, z) = -x\hat{\imath} - y\hat{\jmath} + z^3\hat{k}$, S is the part of the cone $z = \sqrt{x^2 + y^2}$ between $z = 1$ and $z = 3$ with downward orientation. **# 26:** $\vec{F}(x, y, z) = y\hat{\imath} - x\hat{\jmath} + 2z\hat{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$, oriented downward.

[4] **6th ed:** 16.7: [16 of 8th ed], (15), [18 of 8th ed], (18), 19, [24, 26 of 8th ed], (25), 27, (29), (30).

[5] **6th/7th ed:** do the 8th ed problems: **# 2:** $\vec{F}(x, y, z) = y^2z^3\hat{\imath} + 2yz\hat{\jmath} + 4z^2\hat{k}$, E is the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 9$. **# 3:** $\vec{F}(x, y, z) = \langle z, y, x\rangle$, E is the solid ball $x^2 + y^2 + z^2 \leq 16$. **# 11:** $\vec{F}(x, y, z) = (2x^3 + y^3)\hat{\imath} + (y^3 + z^3)\hat{\jmath} + 3y^2z\hat{k}$, S is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.