Math 53 – Practice Midterm 2 B – 90 minutes

Problem 1. (10 points)

a) Draw a picture of the region of integration of $\int_0^1 \int_x^{2x} dydx$.

b) Exchange the order of integration to express the integral in part (a) in terms of integration in the order $dxdy$.

Problem 2. (10 points)

a) Find the mass M of the half-annulus $1 \leq x^2 + y^2 \leq 9$, $y \geq 0$, with density $\rho = \dfrac{y}{x^2+y^2}$.

b) Express the x-coordinate of the center of mass, $\bar{x}$, as an iterated integral. (Write explicitly the integrand and limits of integration.) Without evaluating the integral, explain why $\bar{x} = 0$.

Problem 3. (10 points)

a) Express the work done by the force field $\mathbf{F} = (5x + 3y)\hat{\imath} + (1 + \cos y)\hat{\jmath}$ on a particle moving counterclockwise once around the unit circle centered at the origin in the form $\int_a^b f(t)dt$.
(Do not evaluate the integral; don't even simplify $f(t)$.)

b) Evaluate the line integral using Green's theorem.

Problem 4. (12 points) Consider the rectangle R with vertices $(0,0)$, $(1,0)$, $(1,4)$ and $(0,4)$. The boundary of R is the curve C, consisting of C_1, the segment from $(0,0)$ to $(1,0)$, C_2, the segment from $(1,0)$ to $(1,4)$, C_3 the segment from $(1,4)$ to $(0,4)$ and C_4 the segment from $(0,4)$ to $(0,0)$. Consider the vector field

$$\mathbf{F} = (xy + \sin x \cos y)\hat{\imath} - (\cos x \sin y)\hat{\jmath}$$

a) Find the flux $\int_C \mathbf{F} \cdot \hat{\mathbf{n}}\, ds$ of $\mathbf{F}$ out of R through C. Show your reasoning.

b) Is the total flux out of R through C_1, C_2 and C_3, more than, less than or equal to the flux out of R through C? Show your reasoning.

Problem 5. (12 points) Find the volume of the region enclosed by the plane $z = 4$ and the surface $z = (2x - y)^2 + (x + y - 1)^2$. (Suggestion: change of variables.)

Problem 6. (10 points)

a) Show that the vector field $\mathbf{F} = \langle e^x yz, e^x z + 2yz, e^x y + y^2 + 1\rangle$ is conservative.

b) By a systematic method, find a potential for $\mathbf{F}$.

Problem 7. (12 points) Let S be the part of the spherical surface $x^2 + y^2 + z^2 = 4$, lying in $x^2 + y^2 > 1$, which is to say outside the cylinder of radius one with axis the z-axis.

a) Compute the flux outward through S of the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$.

b) Show that the flux of this vector field through any part of the cylindrical surface $x^2 + y^2 = 1$ is zero.

c) Using the divergence theorem applied to $\mathbf{F}$, compute the volume of the region between S and the cylinder.

Problem 8. (12 points) Consider the surface S given by the equation $z = (x^2 + y^2 + z^2)^2$.

a) Show that S lies in the upper half space ($z \geq 0$).

b) Write out the equation for the surface in spherical coordinates.

c) Using the equation obtained in (b), give an iterated integral, with explicit integrand and limits of integration, which gives the volume of the region inside this surface. Do not evaluate the integral.

Problem 9. (12 points) Let S be the part of the surface $z = xy$ where $x^2 + y^2 < 1$. Compute the flux of $\mathbf{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ upward across S by reducing the surface integral to a double integral.