Math 53 Homework 9

Due Wednesday 10/25/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 10/16: Triple integrals in rectangular coordinates

- **Read:** section 15.6. [7th ed: section 15.7]
- **Work:** 15.6[1]: (3), (9), 13, 15[2], (17), 18, (22), (27), 33, 35.

Wednesday 10/18: Triple integrals in cylindrical coordinates; applications

- **Read:** section 15.7. [7th ed: section 15.8]
- **Work:** 15.6[1]: (39), (43), 46, (53), 54[3]. (feel free to use cylindrical coordinates!)
  15.7[1]: (15), (17), (19), 21, 22, (25), (26), 30.
  Problem 1 below.

Friday 10/20: Triple integrals in spherical coordinates

- **Read:** section 15.8. [7th ed: section 15.9]
- **Work:** 15.8[1]: (5), (7), (9), (13), 14, 15, (17), 19*, (23), 26[4], (29), 30, 33, 35, (39).
  Problem 2 below.

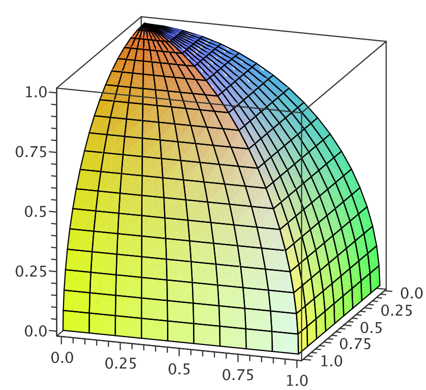
* For 15.8 # 19: set up the integral *both* in cylindrical and in spherical coordinates.

Problem 1.

The picture shows the portion of the solid formed by the intersection of the solid cylinders $y^2+z^2 \leq 1$ and $x^2+z^2 \leq 1$ (two cylinders of radius 1, centered on the x-axis and on the y-axis respectively) which lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$). The front "face" is a portion of the cylinder $x^2+z^2=1$, while the right "face" is part of $y^2+z^2=1$.

Find the volume and the centroid $(\bar{x}, \bar{y}, \bar{z})$ (= center of mass with uniform density $\rho = 1$) of the pictured solid.

(Hint: the integral is easier to set up in the order $dx\,dy\,dz$).



Problem 2. Recall that the *average value* of $f(x,y,z)$ over a region D in space is $\frac{1}{V(D)} \iiint_D f(x,y,z)\,dV$, $\quad V(D) =$ volume of D.
Set up the integral *both* in cylindrical and spherical coordinates for the average distance from a point in the solid sphere of radius a to a point on the surface, and evaluate both integrals. Put the point on the surface at the origin and make it the South pole of the sphere.

---

[1] 7th ed: change 15.6, 15.7, 15.8 to 15.7, 15.8, 15.9.
[2] 7th ed: do 8th ed problem: $\iiint_T y^2\,dV$, T solid tetrahedron with vertices (0,0,0), (2,0,0), (0,2,0) and (0,0,2).
[3] 7th ed: do 8th ed problem: average height of the points in the solid hemisphere $x^2+y^2+z^2 \leq 1$, $z \geq 0$.
[4] 7th ed: do 8th ed problem: $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$, where E lies above the cone $z = \sqrt{x^2+y^2}$ and between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$.