Math 53 Homework 8

Due Wednesday 10/18/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 10/9 – Double integrals in polar coordinates

- **Read:** section 15.3 (7th ed: 15.4).
- **Work:** 15.3[1]: (6), (7), 11, 13, 15, (17), (22), 25, (27), (29), 31[2], (39), 40.

Wednesday 10/11 – Applications of double integrals

- **Read:** section 15.4 (7th ed: 15.5).
- **Work:** 15.4[3]: (3), 10[4], (11), 12 + find the moments of inertia I_x, I_y, I_0, (27), 28.
  Problems 1 and 2 below.

Friday 10/13: Change of variables in double integrals

- **Read:** section 15.9 to the bottom of p. 1058.[5]
- **Work:** 15.9[6]: (1), (3), (7), 15, (17), 19, 23, (24); Problem 3 below.

Problem 1. (The two parts are independent)

a) Show that the average distance of a point in a disk of radius a to the center of the disk is $2a/3$.

b) Find the average distance of a point in a disk of radius a to a fixed point on the circumference of the disk. (Hint: place the center of the disk at $(a, 0)$ and the given point on the circumference at the origin).

Problem 2.

a) Find the area of the region R bounded by the curve $r = \sin 2\theta$ in the first quadrant. (Do this as a double integral in polar coordinates.)

b) Find the coordinates $(\bar{x}, \bar{y})$ of its center of mass (take a uniform density $\rho = 1$). (Hint: it is helpful to rewrite the value of the inner integral as the product of $\sin\theta$ by an expression involving only cosines.)

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[1] 7th ed: 15.4
[2] 7th ed: do the 8th edition problem: $\int_0^{1/2}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\,dx\,dy$.
[3] 7th ed: 15.5
[4] 7th ed: do 8th ed problem: D enclosed by curves $y = 0$ and $y = \cos x$, $-\pi/2 \le x \le \pi/2$; $\rho(x, y) = y$.
[5] 7th ed: section 15.10 to the middle of p. 1046.
[6] 7th ed: 15.10

Problem 3.

Using the coordinate change $u = xy$, $v = y/x$, set up and evaluate an iterated integral for the polar moment of inertia (with density $\rho = 1$) of the region bounded by the hyperbola $xy = 1$, the x-axis, and the two lines $x = 1$ and $x = 2$. Choose the order of integration which makes the limits simplest.