Math 53 Homework 7

Due Wednesday 10/11/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 10/2 – Midterm 1.

The problem below completes the material from Chapter 14, and can be attempted at any time, but is not directly related to the material on Midterm 1.

- **Work:** <u>Problem 1</u> below.

Wednesday 10/4 – Polar coordinates.

- **Read:** section 10.3 through middle of p.663; 10.4 through top of p.671.[1]
- **Work:** <u>Problem 2</u> below.

    10.3: (11), (17), (19), (29), (37), (49), <u>51</u>, (73).

    10.4: (5), <u>7</u>, <u>23</u>[2], (31), <u>35</u>.

Friday 10/6 – Double integrals

- **Read:** sections 15.1, 15.2 (7th ed: 15.1, 15.2, 15.3).
- **Work:** 15.1: (21), <u>24</u>, <u>29</u>, <u>32</u>, (36), (37), (43).[3]

    <u>Problem 3</u> below.

    15.2: (1), (7), (17), (25), <u>27</u>, (39), (46), (48), <u>51</u>, <u>53</u>, (61), <u>64</u>.[4]

Problem 1. – Least-squares interpolation.

In experimental sciences, statistics, and many other fields, one often wishes to establish a linear relationship between two quantities (say x and y). Repeated experiments (or sampling of x and y for various individuals among the general population) give a set of experimental data (x_1, y_1), ..., (x_n, y_n) (each pertaining to a different experiment or to a different individual). One then attempts to find the straight line $y = mx + b$ that best fits the given experimental data. Least-squares interpolation is the most common method for doing so. (Note: the goal is to find m and b, which describe the relation between x and y – we are *not* trying to solve for x and y!). We will first work out the general formula (following a problem in the book), then apply it in an example.

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[1]**7th ed:** 10.3 through middle of p.659; 10.4 through top of p.667.
[2]**7th ed:** do the 8th ed problem: $r = 4 \sin \theta$, $r = 2$.
[3]**7th ed:** 15.2: (9), <u>12</u>, <u>17</u>, <u>20</u>, (24), (25), (31).
[4]**7th ed:** 15.3: (1), (7), (17), (25), <u>27</u>*, (37), (44), (46), <u>49</u>, <u>51</u>**, (59), <u>62</u>.
 * do the 8th edition problem: $2x + y + z = 4$.
 ** do the 8th edition problem: $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3 + 1} \, dy \, dx$.

a) Do section 14.7 exercise # 59 (7th ed: 14.7 # 55). You don't need to prove that the critical point is a minimum.

<u>Hint:</u> In this problem, the total square deviation $\sum_{i=1}^{n} d_i^2$ is a function of the two variables m and b; all the x_i and y_i are constants (as part of the given experimental data). So this is a min-max problem in two variables, just like those we have seen in class. Looking for a critical point should give you the two equations. Proving that the critical point is indeed a minimum can be done (using the second derivative test or other methods) but takes quite a bit of effort; doing it is strictly optional.

b) The table below summarizes recent data for the yearly average CO_2 content of the atmosphere in parts per million (ppm). (Source: NOAA ESRL).

Year	2000	2002	2004	2006	2008	2010	2012	2014	2016
CO_2 (ppm)	369.55	373.28	377.52	381.90	385.60	389.90	393.85	398.65	404.21

We take x to be the year since 2000, and y to be the CO_2 concentration in ppm. So $(x_1, y_1) = (0, 369.55)$, ..., $(x_9, y_9) = (16, 404.21)$.

Use the result of part (a) to write down the equations satisfied by the slope m and the intercept b of the best-fit line for this data. Next, solve these equations and give the values of m and b. (Use a calculator or a computer!)

c) Compare the predicted values $y = mx + b$ with the actual data for 2000, 2008 and 2016. How good is the fit? According to the best-fit line, how much CO_2 will there be in the atmosphere in 2100?

Optional: produce a plot that shows the given data points and the best fit line.

Problem 2. For each of the given curves, find a Cartesian equation for it, and sketch it.

a) $r = 3 \sin \theta$.   b) $r = 5 \sec \theta$.   c) $\theta = -\pi/3$.   d) $r^2 \sin 2\theta = 2$.

Problem 3. Evaluate the following double integrals:

a) $\int_0^1 \int_{x^2}^{x} (1 + 2y) \, dy \, dx$,

b) $\iint_D \dfrac{y}{x^5 + 1} \, dA$, $D = \{(x, y) \,|\, 0 \le x \le 1, \ 0 \le y \le x^2\}$,

c) $\iint_D 2xy \, dA$, D is the triangular region with vertices (0,0), (1,2), and (0,3).