Math 53 Homework 5

Due Wednesday 9/27/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 9/18 – Chain rule

- **Read:** section 14.5.
- **Work:** 14.5: (1), 5, 7[1], (13), 15, (17), 18[2], (23), (39), 43, 45, (47), 51, (53).

Wednesday 9/20 – Gradient, directional derivatives

- **Read:** section 14.6.
- **Work:** 14.6: 1, (7), 9, (11), (21), 27, (37), 38, 41, (43), 49, (51), 56.
  Problems 1, 2 and 3 below.

Friday 9/22 – Partial differential equations; max-min problems

- **Read:** section 14.3 p.920-921 [7th ed: p.908-909]; section 14.7.
- **Work:** 14.3: 75, (76), (80), 81[3]
  14.7: (1), 3, (7), 11, (23), (39), 43, (47), 49.[4]

Problem 1. a) We consider again the contour plot of the function $f(x, y)$ depicted in HW 4 Problem 2. Imagine that we move from the point $(\frac{3}{2}, \frac{1}{2})$ along a straight line whose direction is given by a unit vector $\langle a, b\rangle$, so $x(t) = \frac{3}{2}+at$ and $y(t) = \frac{1}{2}+bt$, and consider the value of $f(x(t), y(t))$ as a function of t.

(i) What are the possible direction(s) $\langle a, b\rangle$ for which $df/dt = 0$ at $t = 0$? (Either sketch a picture showing a portion of level curve and the vector $\langle a, b\rangle$, or give approximate angle from the x-axis). For each of these directions, when travelling along the chosen line, does the value of f pass through a minimum or a minimum at $t = 0$?

(ii) What are the direction(s) $\langle a, b\rangle$ for which df/dt at $t = 0$ is largest, resp. smallest? Measure the contour plot to estimate the values of df/dt at $t = 0$ for those directions.

b) Recall that the contour plot depicted in HW 4 Problem 2 corresponds to $f(x, y) = x(x - 1)(x - 2) + (y - 1)(x - y)$. At $(\frac{3}{2}, \frac{1}{2})$:

(i) find the unit vectors $\langle a, b\rangle$ such that the derivative of f along the line $x(t) = \frac{3}{2}+at$, $y(t) = \frac{1}{2} + bt$ is zero at $t = 0$.

(ii) find the unit vectors $\langle a, b\rangle$ such that the derivative of f along the line $x(t) = \frac{3}{2} + at$, $y(t) = \frac{1}{2} + bt$ is largest, resp. smallest at $t = 0$, and calculate the value of df/dt at $t = 0$ for those directions.

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[1] **7th ed:** do the 8th ed version: $z = (x - y)^5$, $x = s^2t$, $y = st^2$.

[2] **7th ed:** do the 8th ed version: $w = f(x, y, z)$, where $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$.

[3] **7th ed:** do the 8th ed version: show that $c(x, t) = (4\pi Dt)^{-1/2}e^{-x^2/(4Dt)}$ is a solution of the diffusion equation $\partial c/\partial t = D\,\partial^2 c/\partial x^2$.

[4] **7th ed:** 14.7: (1), 3, (7), 11 with $f(x, y) = x^3 - 3x + 3xy^2$, (21), (37), 41, (45), 47.

Problem 2.

Find the maximum rate of change of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 6, -2)$, and the direction in which it occurs. Optional: find a geometric interpretation to your answer.

Problem 3.

a) Find the direction from $(4, 2, 3)$ in which $g(x, y, z) = x^2 + y^2 - 6z$ decreases fastest.

b) Follow the line in the direction you found in part (a) to estimate, using linear approximation, the location of the point closest to $(4, 2, 3)$ at which $g = 0$. Do not use a calculator. Express your answer using fractions. Next, use a calculator to evaluate g at your point. (The value should be reasonably small.)

(Hint: in the linear approximation, the closest point where $g = 0$ lies on the line through $(4, 2, 3)$ in the direction found in (a). Find a parametric equation for this line, then use linear approximation to estimate the value of g at a point on the line.)