

Math 53 Homework 4

Due Wednesday 9/20/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 9/11 – Functions of several variables

• **Read:** sections 14.1; also 12.6 to the bottom of p. 837¹; and 14.2 (skip the theory, focus on examples 1–3, the definition of continuity, examples 7–8).

• **Work:** Problem 1 below.

14.1: (13), (25), (32), 36, (41), (47), 61, (63), 68, (72), (80).²

14.2: (4), (7), 9, 13, (27), (33), 39, (43).

Wednesday 9/13 – Partial derivatives, tangent plane, linear approximation

• **Read:** section 14.3 to bottom of p. 921.³

• **Work:** 14.3: (5), 10, (11), 24, (37), (41), (42), 47, (51), (53), (61), 69, 77, 79, 88, 97.⁴

Problems 2 and 3 (next page)

Friday 9/15 – Tangent plane, linear approximation

• **Read:** section 14.4[†].

† PLEASE *don't mix differentials like dz with numerical differences like Δx or Δy . Statements such as “ $dx = \Delta x$ ” are to be avoided. See lecture.*

• **Work:** 14.4: (1), 5, 18⁵, (19), 21, (25), 28, 33[†], 38[†], (39).

Problem 4 (next page).

† *Whenever the book says “use differentials to estimate ...”, read “use linear approximation to estimate ...”.*

Problem 1.

a) Sketch the graph of the function $f(x, y) = 3 - x^2 - y^2$.

b) Sketch the graph of the function $g(x, y) = \sqrt{x^2 + y^2}$.

c) Make a rough sketch of a contour plot for the function whose graph is depicted in Figure 10(a) on page 892 [7th ed: p. 882].

d) Draw a contour plot of the function $f(x, y) = e^{y/x}$ showing several level curves.

Problem 2. The figure next page is the contour plot of a function of two variables $f(x, y)$, for x and y ranging between 0 to 2 (scale: 1 unit = 5 cm; spacing between contour levels: 0.2).

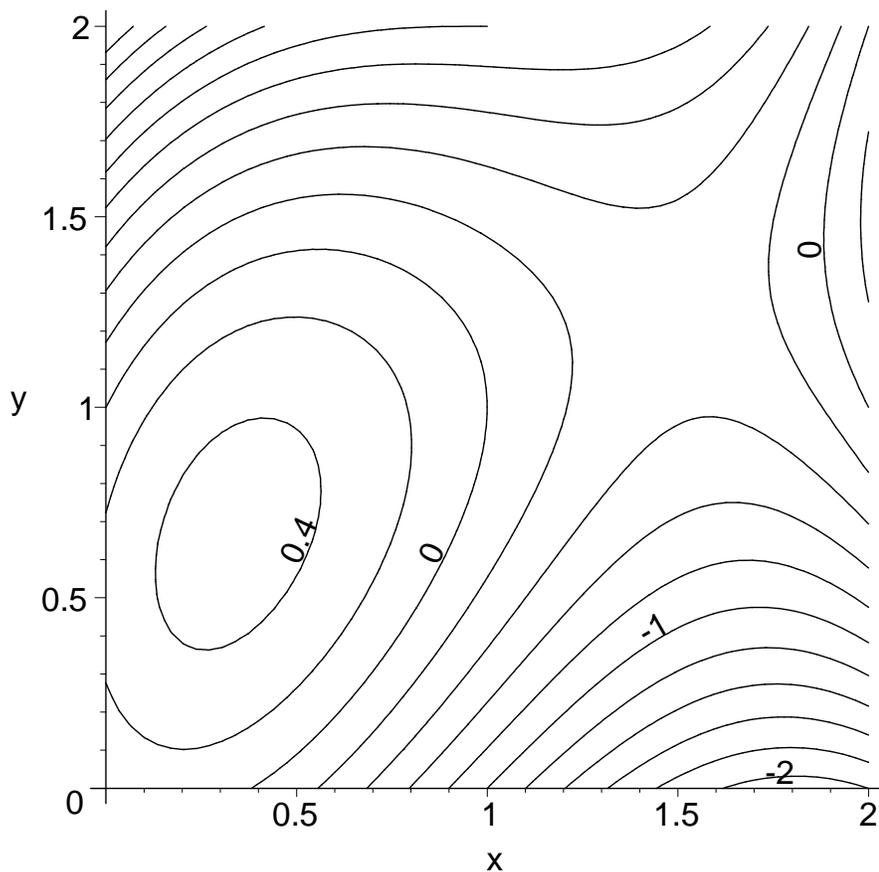
¹7th ed: bottom of p. 830

²7th ed: 14.1: (13), (25), (32), 36, (39), (45), 59, (61), 66, (70), (78).

³7th ed: bottom of p. 909.

⁴7th ed: 14.3: (5), 10, (11), 24, (37), (41), (42), 47, (51), (53), (61), 69, 77, 79, 88, 93.

⁵7th ed: do the 8th ed version, $(y - 1)/(x + 1) \approx x + y - 1$.



- a) Use the contour plot to determine whether f_x and f_y are > 0 , $= 0$, or < 0 at the point $(\frac{3}{2}, \frac{1}{2})$. Same question at the point $(1, 1)$?
- b) Use the contour plot to find two points where $f_x = f_y = 0$, and give their approximate coordinates. What happens to the level curves of f through these points? For each of the two points, describe what happens when you move towards North, South, East, West: does the value of f go up, down, or does it stay exactly the same?

Problem 3.

The function in the previous problem is $f(x, y) = x(x - 1)(x - 2) + (y - 1)(x - y)$.

- a) Calculate the actual values of the partial derivatives at $(\frac{3}{2}, \frac{1}{2})$ and $(1, 1)$.
- b) Find the points where $f_x = f_y = 0$, and calculate the second partial derivatives f_{xx} and f_{yy} at these points. Relate your answer to your findings in Problem 2(b).

Problem 4.

Assume that $b^2 - 4c > 0$ so that $x^2 + bx + c = 0$ has two roots. Let r denote the larger root. Then r is a function of b and c .

- a) Give an approximate formula for the small change Δr in the value of r produced by small changes Δb and Δc in the coefficients. Use this to calculate an approximate value for the larger root of $x^2 - 5.01x + 3.98 = 0$. Compare your answer with the exact value.
- b) Starting from the equation $x^2 - 5x + 4 = 0$, is r more sensitive to small changes in b or c ? (Justify your answer.)