Math 53 Homework 3

Due Wednesday 9/13/17 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Wednesday 9/6 – Parametric equations: velocity, acceleration

- **Read:** sections 10.2 to end of Example 2; 13.2; 13.4 to p. 873 [7th ed: p. 865].
- **Work:** 10.2: (3), 7, (17), 73.
  13.2: (1), (3), 5[1], (9), (19), 25, 33, (34).
  13.4: (3), 10, (15).

Friday 9/8 – Parametric equations: arc length; further examples

- **Read:** sections 10.2 to end of Example 5; 13.2; 13.3 p. 830-831.[2]
- **Work:** 10.2: (31), 32, (33), 41, (43), (51), (53).
  Problems 1, 2, 3 (below).
  13.2: (41), (47), 53, 55, 56.[3]

Problem 1. Consider the parametric curve given by: $x = 2\cos t$, $y = \sin 2t$.

a) Find the points on the curve where the tangent is horizontal or vertical.

b) Show that the curve has two tangents at $(0,0)$ and find their equations.

c) Sketch the curve.

d) Calculate the total area enclosed by the curve.

Problem 2. One circle has radius a and center at the origin. A second circle of same radius a has a point P marked on it, which is initially at $(a,0)$. The second circle rolls without slipping counterclockwise around the first, until it has returned to its starting position.

a) Write parametric equations for the motion of P, using as parameter θ, the angle by which the contact point has turned.

(Hint: first find the position of the center of the moving circle; then determine the distance and direction from the center to P. If you have trouble visualizing the problem, try it out experimentally with two coins).

b) Find the length of the trajectory of P. (Hint: the integrand simplifies to a constant times $\sqrt{2-2\cos\theta}$; use the half angle $\theta/2$ for further simplification.)

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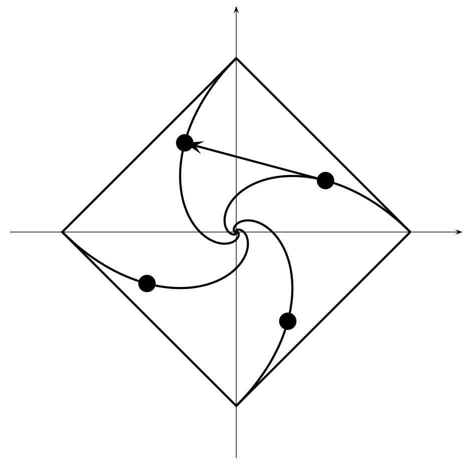
[1] **7th ed:** do the 8th ed problem: $\vec{r}(t) = e^{2t}\hat{\imath} + e^{t}\hat{\jmath}$, $t = 0$.
[2] **7th ed:** 13.3 to middle of p. 855.
[3] **7th ed:** 13.2: (41), (47), 51, 53, 54.

Problem 3.

Four bugs are placed at the four corners of the square with vertices at $(\pm 1, 0)$ and $(0, \pm 1)$. The bugs crawl counterclockwise, with each bug crawling directly toward the next bug at all times, at a speed equal to the distance separating the two bugs. They approach the center of the square along spiral paths.



Denote by $\vec{r}(t) = \langle x(t), y(t) \rangle$ the position vector of the bug that starts at $(1, 0)$.

a) Show that the velocity vector is $\vec{v}(t) = \langle -x(t) - y(t), x(t) - y(t) \rangle$.
(Hint: use symmetry to find the position of the bug that starts at $(0, 1)$.)

b) Calculate $d(|\vec{r}|^2)/dt$, and solve for $|\vec{r}(t)|$ as a function of t.

c) Denoting by $\theta(t)$ the angle between the x-axis and $\vec{r}(t)$, express $\vec{r}(t)$ and $\vec{v}(t)$ in terms of $\theta(t)$ and its derivative. Then compare your expression for $\vec{v}(t)$ with that found in (a) in order to determine $d\theta/dt$. Finally, solve for $\theta(t)$ and $\vec{r}(t)$.

d) Find the total distance traveled by a bug over time (all the way from its initial position to the center of the square).