Math 53 – Practice Midterm 1 B – 90 minutes

Problem 1.

Let P, Q and R be the points at 1 on the x-axis, 2 on the y-axis and 3 on the z-axis, respectively.

a) (4) Express $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in terms of $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$.

b) (4) Find the cosine of the angle PQR.

Problem 2. Let $P = (1, 1, 1)$, $Q = (0, 3, 1)$ and $R = (0, 1, 4)$.

a) (6) Find the area of the triangle PQR.

b) (3) Find the plane through P, Q and R, expressed in the form $ax + by + cz = d$.

c) (3) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to the plane in part (b)? Explain why or why not.

Problem 3. A ladybug is climbing on a Volkswagen Bug (= VW). In its starting position, the the surface of the VW is represented by the unit semicircle $x^2 + y^2 = 1$, $y \geq 0$ in the xy-plane. The road is represented as the x-axis. At time $t = 0$ the ladybug starts at the front bumper, $(1, 0)$, and walks counterclockwise around the VW at unit speed relative to the VW. At the same time the VW moves to the right at speed 10.

a) (7) Find the parametric formula for the trajectory of the ladybug, and find its position when it reaches the rear bumper. (At $t = 0$, the rear bumper is at $(-1, 0)$.)

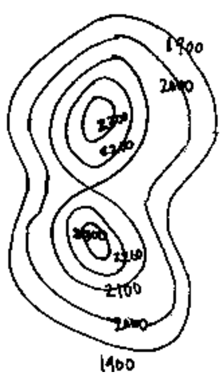
b) (5) Compute the speed of the bug, and find where it is largest and smallest. Hint: It is easier to work with the square of the speed.

Problem 4.

(a) (4) Let $P(t)$ be a point with position vector $\vec{r}(t)$. Express the property that $P(t)$ lies on the plane $4x - 3y - 2z = 6$ in vector notation as an equation involving $\vec{r}$ and the normal vector to the plane.

(b) (4) By differentiating your answer to (a), show that $\frac{d\vec{r}}{dt}$ is perpendicular to the normal vector to the plane.

Problem 5. (5) On the contour plot below, mark the portion(s) of the level curve $f = 2000$ on which $\frac{\partial f}{\partial y} \geq 0$.



Problem 6. Let $f(x, y) = x^2y^2 - x$.

a) (4) Find ∇f at $(2, 1)$

b) (2) Write the equation for the tangent plane to the graph of f at $(2, 1, 2)$.

c) (2) Use a linear approximation to find the approximate value of $f(1.9, 1.1)$.

d) (2) Find the directional derivative of f at $(2, 1)$ in the direction of $-\hat{\imath} + \hat{\jmath}$.

Problem 7. a) (7) Find the critical points of

$$w = -3x^2 - 4xy - y^2 - 12y + 16x$$

and say what type each critical point is.

b) (8) Find the point of the first quadrant $x \geq 0$, $y \geq 0$ at which w is largest. Justify your answer.

Problem 8. Let $u = y/x$, $v = x^2 + y^2$, $w = w(u, v)$.

a) (5) Express the partial derivatives w_x and w_y in terms of w_u and w_v (and x and y).

b) (3) Express $xw_x + yw_y$ in terms of w_u and w_v. Write the coefficients as functions of u and v.

c) (2) Find $xw_x + yw_y$ in case $w = v^5$.

Problem 9. a) (7) Find the Lagrange multiplier equations for the point of the surface

$$x^4 + y^4 + z^4 + xy + yz + zx = 6$$

at which x is largest. (Do not solve.)

b) (3) Given that x is largest at the point (x_0, y_0, z_0), find the equation for the tangent plane to the surface at that point.

Problem 10. Suppose that $x^2 + y^3 - z^4 = 1$ and $z^3 + zx + xy = 3$.

a) (5) Take the total differential of each of these equations.

b) (5) The two surfaces in part (a) intersect in a curve along which y is a function of x. Find dy/dx at $(x, y, z) = (1, 1, 1)$.