Math 277 Section 3 – Homework 1 – due Thursday October 29, 2009.

This homework assignment will be graded leniently; what matters most to me is not completeness of your solution, but evidence that you've spent some time thinking seriously about the questions.

1. Let $p \in H^4(\mathbb{CP}^2)$ and $\ell \in H^2(\mathbb{CP}^2)$ be the cohomology classes Poincaré dual to a point and a line respectively. Calculate the Gromov-Witten invariant $\langle p, p, \ell \rangle_{0,1}$ counting degree 1, genus 0 curves in $\mathbb{CP}^2$.

(There are various ways of doing this, but you will need to address the question of whether the moduli space you consider is regular).

2. Consider the family of cubic elliptic curves $C_t : \{x_0^3 + x_1^3 + x_2^3 - t x_0 x_1 x_2 = 0\}$ in $\mathbb{CP}^2$. Construct a holomorphic 1-form on C_t, and find the Picard-Fuchs differential equation satisfied by its periods.

Optional: calculate the first few terms in the expansion of the periods, and use this to calculate the beginning of the asymptotic expansion for the canonical coordinate near the large complex structure limit $t \to \infty$.

(Note: use the case of the quintic 3-fold as inspiration!)