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<u>Recall:</u> quintic mirror family $\check{X}_\psi$, with LCSL degeneration as $z=(5\psi)^{-5}\to 0$

We computed the canonical coordinate $q=\exp(2\pi i w)$, $w=\dfrac{\int_{\beta_1}\check{\Omega}}{\int_{\beta_0}\check{\Omega}}$

and the Yukawa coupling

$$\left\langle \frac{\partial}{\partial w}, \frac{\partial}{\partial w}, \frac{\partial}{\partial w}\right\rangle = \frac{1}{\left(\int_{\beta_0}\check{\Omega}\right)^2}\int_{\check{X}} \check{\Omega}\wedge\frac{\partial^3\check{\Omega}}{\partial w^3}$$

$$= -c_1 - 575\frac{c_1}{c_2}q - \frac{1950750}{2}\frac{c_1}{c_2^2}q^2 - \frac{10277490000}{6}\frac{c_1}{c_2^3}q^3 - \frac{7448604862500 0}{24}\frac{c_1}{c_2^4}q^4+\dots$$

- Now recall: under mirror symmetry, expect $\exists$ basis $\{e\}$ of $H^2(X,\mathbb{Z})\simeq\mathbb{Z}$ (i.e.: $e=PD(\text{hyperplane})$) s.t., writing $[B+i\omega]=te$, $q=\exp(2\pi i t)=e^{2\pi i\int_{\text{line}}B+i\omega}$

Then the mirror map is $q\longleftrightarrow q$

I.e. $w=\frac{1}{2\pi i}\log q \leftrightarrow t$,

$\frac{\partial}{\partial w}\leftrightarrow\frac{\partial}{\partial t}$ $\leftarrow$ <u>this</u> is the identification we want between $H^{2,1}(\check{X})\simeq H^{1,1}(X)$

(Rather: $\frac{\partial}{\partial t}\in T\mathcal{M}_{\text{käh}} \leftrightarrow \frac{\partial}{\partial t}[B+i\omega]=e\in H^{1,1}$).

Now on sympl. side the Yukawa coupling was

$$\left\langle\frac{\partial}{\partial t},\frac{\partial}{\partial t},\frac{\partial}{\partial t}\right\rangle = \langle e,e,e\rangle = \int_X e\wedge e\wedge e + \sum_{\substack{d>0\\ \text{degree}}} \underbrace{\langle e,e,e\rangle_{0,d}}_{\text{GW invt}}\, q^d$$

$$\text{where } \langle e,e,e\rangle_{0,d} = \underbrace{\left(\int_d e\right)\left(\int_d e\right)\left(\int_d e\right)}_{=d^3} N_d,\quad N_d=\sum_{d=kd'}\frac{n_{d'}}{k^3}$$

$$\Rightarrow \langle e,e,e\rangle = 5+\sum_{d=1}^{\infty} d^3 N_d q^d = 5+\sum_{d=1}^{\infty} d^3 n_d\frac{q^d}{1-q^d}$$

$$= 5+n_1 q+8\left(n_2+\frac{n_1}{8}\right)q^2+27\left(n_3+\frac{n_1}{27}\right)q^3+64\left(n_4+\frac{n_2}{8}+\frac{n_1}{64}\right)q^4+\dots$$

Match with above $\Rightarrow$
- $c_1=-5$
- $n_1=\dfrac{575.5}{c_2}=\dfrac{2875}{c_2}$

In fact, classical alg. geom. $\Rightarrow n_1=2875$, so $c_2=1$.

② Then get the others: • $n_2 = 609250$ (had been obtained by S. Katz 1986)

• $n_3 = 317206375$ (checked by Ellingsrud-Strømme 1990)

• $n_4 = 242467530000$

General verification (by pf of mirror symmetry for quintic in sense stated here) by Givental and Lian-Liu-Yau ~1996.

— more generally, for CY obtained as complete intersections in toric varieties.

We'll now switch to a different mathematical formulation of mirror symmetry, due to Kontsevich ~1994: <u>homological mirror symmetry</u>

- On the symplectic side, we used to look at J-holomorphic spheres to get a "quantum" version of intersection pairing on $H^*(X)$, now we'll look at intersections of Lagrangian submanifolds, and "quantum" intersection theory involving J-holomorphic disks with boundary on Lagrangians
- On the complex side, we'll look at $\begin{cases} \text{intersections of subvarieties} \\ \text{holomorphic } \begin{matrix}\text{maps} \\ \text{extensions}\end{matrix} \text{ of } \begin{matrix}\text{bundles} \\ \text{sheaves}\end{matrix} \end{cases}$

We'll outline briefly the constructions of:

- the Fukaya (A_∞)-category (roughly: objects = Lagr. submflds
  morphisms = intersections
  alg. structure (diff!, product, ...) = hol. disks)
- the cat. of coherent sheaves

HMS states the corresponding <u>derived categories</u> are equivalent

We'll try to understand a simple example = T^2

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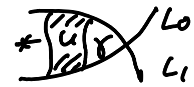
③ Lagrangian Floer homology:

(M,ω) symplectic manifold $\supset L_0, L_1$ compact Lagrangian submanifolds

Formally, Floer homology = Morse theory for "action functional" on path space $\mathcal{P}(L_0,L_1)$, whose crit pts are constant paths & gradient flowlines = J-hol. strips.

More precisely: $\underset{\text{univ. cover}}{\mathcal{A}: \widetilde{\mathcal{P}}(L_0,L_1) \to \mathbb{R}}$, $(\gamma,[u])$  $\mapsto \int u^*\omega$

$u: [0,1]^2 \to M$ homotopy $* \to \gamma$

$$d\mathcal{A}(\gamma)\cdot v = \int_{[0,1]} \omega(\dot\gamma, v)\,dt = \int_{[0,1]} g(J\dot\gamma, v)\,dt = \langle J\dot\gamma, v\rangle_{L^2}$$

v: vect. field along γ, $v(0) \in T_{\gamma(0)}L_0$, $v(1) \in T_{\gamma(1)}L_1$

hence crit pts = const. paths $\dot\gamma = 0$; gradient trajs. = J-hol. maps $\frac{\partial\gamma}{\partial s} = -J\dot\gamma$

Difficult to define rigorously ∞-dim'l Morse theory, so use holom. curves instead.