

Math 112 Homework 2

Due Thursday February 14, 2019, on Canvas.

You are encouraged to discuss the homework problems with other students. However, what you hand in should reflect your own understanding of the material. You are NOT allowed to copy solutions from other students or other sources. *No late homeworks will be accepted.*

Please pay attention to the clarity and precision of your answers. Your solutions to the problems should always consist of carefully written mathematical arguments.

Material covered: Rudin pages 27–36.

Problem 1. (5 points) Rudin Chapter 2 Problem 2.

Problem 2. (4 points)

For $x, y \in \mathbb{R}$, define: $d_1(x, y) = |x - y|^2$, $d_2(x, y) = |x^2 - y^2|$, $d_3(x, y) = \frac{|x - y|}{1 + |x - y|}$.

Determine for each of these functions whether it defines a metric on \mathbb{R} .

Problem 3. (6 points)

(a) If $p = (p_1, \dots, p_n)$ and $q = (q_1, \dots, q_n)$ are points in \mathbb{R}^n , define

$$d_\infty(p, q) = \sup\{|p_i - q_i|, i = 1, 2, \dots, n\}.$$

Show that d_∞ is a metric on \mathbb{R}^n .

(b) Let $d(p, q) = |p - q|$ be the standard metric on \mathbb{R}^n . Prove that E is open in (\mathbb{R}^n, d) if and only if E is open in (\mathbb{R}^n, d_∞) .

Problem 4. (4 points)

Let X be any set. For $p, q \in X$ define $d(p, q) = \begin{cases} 1 & \text{if } p \neq q, \\ 0 & \text{if } p = q. \end{cases}$

Prove that this is a metric. Which subsets of the metric space (X, d) are open? Which are closed?

Problem 5. (3 points) Let E' be the set of all limit points of a set E . Prove that E' is closed.

Problem 6. (4 points) Rudin Chapter 2 Problem 7 (Hint: use Theorem 2.27).