

# Math 112 Homework 1

Due Thursday February 7, 2019, on Canvas.

You are encouraged to discuss the homework problems with other students. However, what you hand in should reflect your own understanding of the material. You are NOT allowed to copy solutions from other students or other sources. *No late homeworks will be accepted.*

Please pay attention to the clarity and precision of your answers. Your solutions to the problems should always consist of carefully written mathematical arguments.

Be especially careful about making the structure of your argument understandable: use sentences (“Assume that ...”, “We want to prove that ...”, “Therefore, ...”) to clarify the status of the various mathematical assertions in your answer.

**Material covered:** Rudin pages 1-17, 24-30.

1. (3 points) Rudin Chapter 1 Problem 1.

2. (5 points) Rudin Chapter 1 Problem 5.

3. (5 points) Rudin Chapter 1 Problem 9.

4. (5 points)

(a) Let  $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ . Prove that  $\mathbb{Q}(\sqrt{2})$  is a subfield of  $\mathbb{R}$ .

(Hint: observe that, since  $\mathbb{Q}(\sqrt{2})$  is contained in  $\mathbb{R}$ , it is enough to check that the operations  $x + y$ ,  $-x$ ,  $xy$ , and  $1/x$  are well-defined in  $\mathbb{Q}(\sqrt{2})$  – explain why).

(b) Prove that  $\sqrt{3}$  does not belong to  $\mathbb{Q}(\sqrt{2})$ .

(Hint: compute the square of  $a + b\sqrt{2}$  and argue by contradiction).

5. (3 points)

Assume that  $n \geq 2$  is an integer and  $z_1, z_2, \dots, z_n$  are complex numbers such that  $z_1 + z_2 + \dots + z_n = 0$  and  $|z_1| = |z_2| = \dots = |z_n| = 1$ . Prove that

$$|1 + z_1|^2 + |1 + z_2|^2 + \dots + |1 + z_n|^2 = 2n.$$

6. (4 points) Rudin Chapter 1 Problem 18.

7. (5 points) Let  $\mathbb{Z}$  be the set of all integers, and  $M = \{S \subset \mathbb{Z}, S \text{ is finite}\}$  the set of all finite subsets of  $\mathbb{Z}$ . Show that  $M$  is countable.

(Hint: show that  $M$  can be described as the union of countably many finite or countable sets).

8. How long did this assignment take you? How hard was it? What resources did you use, and how much help did you need? Did you have any prior experience with abstract math and writing mathematical arguments?