

Theorem. Let K/\mathbb{Q} be a number field and $I \subseteq \mathfrak{o}_K$ an ideal. Then: there are $x, y \in I$ such that $I = (x, y)$.

Proof. Let¹ $\mathfrak{p} \neq \mathfrak{q} \subseteq \mathfrak{o}_K$ be primes of \mathfrak{o}_K with ideal class that of I^{-1} , and note that $I = I \cdot (\mathfrak{p} + \mathfrak{q}) = I \cdot \mathfrak{p} + I \cdot \mathfrak{q}$.

¹(— via Chebotarev applied to the Hilbert class field of K)