

Theorem. Let G be a connected compact Lie group and $T, T' \subseteq G$ maximal tori. Then: there is a $g \in G$ with $T' = g^{-1} \cdot T \cdot g$.

Proof. Let $t \in T$ and $t' \in T'$ be topological generators, and note that $t \curvearrowright G/NT$ fixes only the identity coset — thus by homotopy invariance $\pm 1 = \sum_i (-1)^i \cdot \text{tr}(t' \curvearrowright H^i(G/NT, \mathbb{Q}))$, so that $t' \curvearrowright G/NT$ has a fixed point, e.g. $g \cdot NT$, whence $T' = g^{-1} \cdot T \cdot g$ since $(NT)^\circ = T$.