Solutions to Trig Review Problems

① Two high tides and two low tides every 25 hours gives a period of $25/2$ hours.

Since we want to model the tidal level with a function like

$$f(x) = A \sin Bx$$

we have that

$$\text{period} = \frac{2\pi}{B}$$

$$\Rightarrow \frac{25}{2} = \frac{2\pi}{B} \Rightarrow B = \frac{4\pi}{25}$$

Since the tidal range is 45 feet, $A = \frac{45}{2}$.

$$\Rightarrow f(x) = \frac{45}{2} \sin\left(\frac{4\pi}{25} x\right),$$

where $f(x)$ gives the height in feet above or below sea level x hours after a given "level" tide.

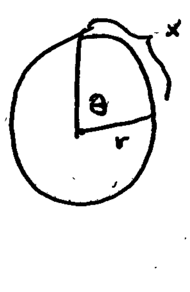
② (a) $x = n\pi$, where n is any integer

(b) $x = \frac{\pi}{2} + n\pi$, where n is any integer

(c) Since sine is 2π-periodic,

$$\sin(x + 2\pi) = \sin x$$

(d) Likewise, $\cos(x - 6\pi) = \cos x$.

③ (a) 

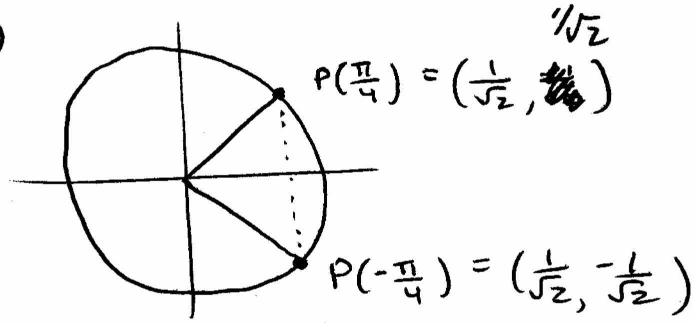
$$\frac{x}{\text{circumference}} = \frac{\theta}{2\pi}$$

$$\Rightarrow \frac{x}{2\pi r} = \frac{\theta}{2\pi} \Rightarrow \boxed{x = \theta r}$$

(b) $\dfrac{x}{2\pi r} = \dfrac{\theta}{360} \Rightarrow \boxed{x = \dfrac{\theta \pi r}{180}}$

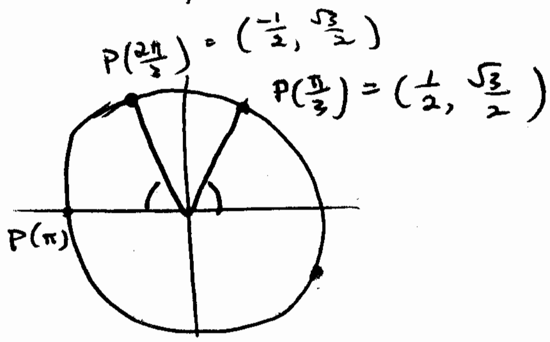
Note that the radian version of the formula is much simpler.

④ (a)



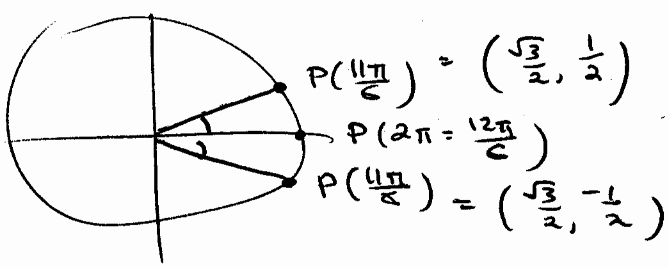
$\Rightarrow \sin(-\frac{\pi}{4}) = \frac{-1}{\sqrt{2}}$

(b)



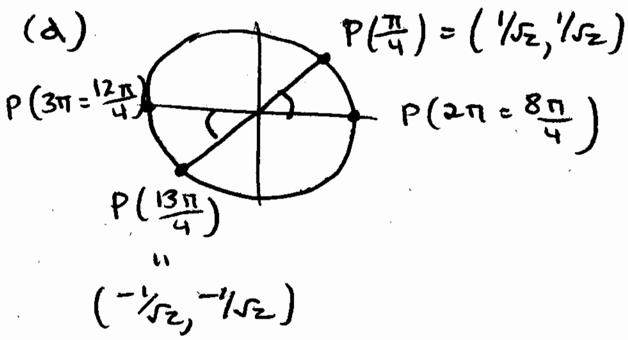
$\Rightarrow \cos(\frac{2\pi}{3}) = \frac{-1}{2}$

(c)



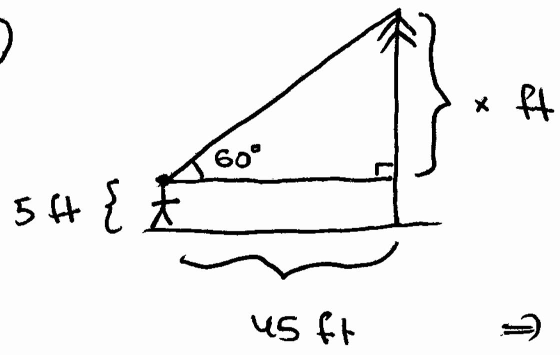
$\Rightarrow \tan\frac{11\pi}{6} = \dfrac{-1/2}{\sqrt{3}/2} = \dfrac{-1}{\sqrt{3}}$

(d)



$\Rightarrow \sin(\frac{13\pi}{4}) = -1/\sqrt{2}$

(5)



$$\tan 60^\circ = \frac{\text{opp}}{\text{adj}} = \frac{x}{45}$$

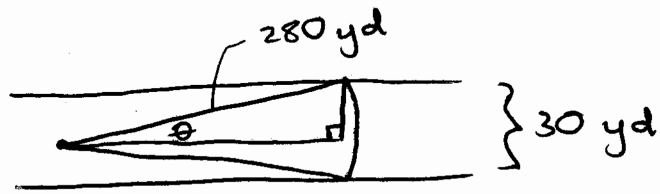
$$x = 45 \tan 60^\circ$$

$$= 45 \tan \frac{\pi}{3} = 45\sqrt{3} \text{ ft}$$

$\Rightarrow$ The tree is

$5 + 45\sqrt{3} \simeq 82.94$ ft tall.

(6)



$$\sin\theta = \frac{15}{280} \Rightarrow \theta = \sin^{-1}\left(\frac{15}{280}\right) \simeq .0536 \text{ rad}$$

$$\simeq 3.07^\circ$$

* Note that we need a calculator for this one since none of the special angles we know has a sine value of $15/280$.

(7) (a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pi/3$

(b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = 3\pi/4$

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The angle between 0 and $\frac{\pi}{4}$ with cosine $-1/\sqrt{2}$

(c) $\tan^{-1} 1 = \pi/4$

(d) $\tan^{-1}(-1)$ = angle between $-\pi/2$ and $\pi/2$ with tangent -1 = $-\pi/4$

8 (a) $2\cos x + \tan x = \sec x$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$$\Rightarrow 2\cos x + \frac{\sin x}{\cos x} = \frac{1}{\cos x}$$

$$\Rightarrow 2\cos^2 x + \sin x = 1$$

$$\Rightarrow 2(1-\sin^2 x) + \sin x = 1$$

$$\Rightarrow 0 = 2\sin^2 x - \sin x - 1$$

$$\Rightarrow 0 = (2\sin x + 1)(\sin x - 1)$$

$2\sin x + 1 = 0$ OR $\sin x - 1 = 0$

$\sin x = -1/2$ $\qquad$ $\sin x = 1$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$ or $\boxed{x = \pi/2}$

(b) $\sin x \cos 2x + \cos x \sin 2x = 0$

$\sin x \cos 2x + \cos x (2\sin x \cos x) = 0$

$\sin x \cos 2x + 2\sin x \cos^2 x = 0$

$\sin x (2\cos^2 x - 1) + 2\sin x \cos^2 x = 0$

I use this $\cos 2x$ identity so that I'll get another $\sin x \cos^2 x$ term.

$2\sin x \cos^2 x - \sin x + 2\sin x \cos^2 x = 0$

$4\sin x \cos^2 x - \sin x = 0$

$\sin x (4\cos^2 x - 1) = 0$

$\sin x = 0$ OR $4\cos^2 x - 1 = 0$

$\boxed{x = 0, \pi}$

$\cos^2 x = 1/4$

$\cos x = \pm 1/2$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

(c) $\sin\left(3x - \frac{\pi}{4}\right) = 1$

$$3x - \frac{\pi}{4} = \ldots, -\frac{7\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$$

$$3x = \ldots, -\frac{13\pi}{4}, -\frac{5\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{4}, \frac{19\pi}{4}, \ldots$$

$$x = \ldots, -\frac{13\pi}{12}, -\frac{5\pi}{12}, \underbrace{\frac{3\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12},}_{\text{These are the only solutions in } [0, 2\pi).} \frac{27\pi}{12}, \ldots$$