$$
\begin{array}{ccccccccc}
 & & \pi^* V_d^g & & V_d^g & & \rho^* V_d^g & & V \\
 & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
W_d & \stackrel{\phi}{\longleftarrow} & M_d^g(X) & \stackrel{\pi}{\longrightarrow} & \mathcal{M}_{g,0}(d,X) & \stackrel{\rho}{\longleftarrow} & \mathcal{M}_{g,1}(d,X) & \stackrel{ev}{\longrightarrow} & X
\end{array}
$$

$$
\begin{aligned}
W_d &= \text{linearized moduli}, \\
M_d^g(X) &= \text{nonlinear moduli}, \\
\mathcal{M}_{g,0}(d,X) &= \text{moduli space of stable maps} \\
\text{fiber of } V_d^g &= H^0(C, f^*V) \oplus H^1(C, f^*V) \\
\omega &= \pi^* b(V_d^g),
\end{aligned}
$$

where b is a multiplicative characteristic class

$$Q_d = \phi_!(\omega)$$