Historical context of the Banu Musa treatise Connemara Doran, HS 206r

The Banu Musa (the sons of Musa ibn Shakir, an astronomer/astrologer) were three mathematician brothers (Muhammad, Ahmad, and Hasan) whose works as members of the newly-established House of Wisdom (in 9th century Baghdad) helped preserve and advance ancient Greek mathematics among Islamic mathematicians. One of their works, the *Verba filiorum* – a short treatise based on Archimedes' *On the Sphere and the Cylinder* – contributed directly as well to mathematical advancement in the West due to the wide (long-lasting) dissemination of a 12th century Latin translation by Gerard of Cremona. Clagett's translation is based on the Arabic text as well as a complete early 14th century manuscript copy (P) of Gerard's Latin translation. Clagett was aware of eight manuscripts of the translation.

The House of Wisdom brought together (all appointed by the Caliph al-Ma'mum) a group of talented translators and mathematicians who studied Greek scientific manuscripts brought to Baghdad from Byzantium and its provinces. There has been some debate regarding whether the *Verba filiorum* was influenced by another mathematician-translator Thabit ibn Qurra. An interesting question is what stimulated the new approaches taken by the Banu Musa. The mathematician al-Khwarizmi (c. 780-850) was a (slightly older) contemporary of the Banu Musa at the House of Wisdom. He is famous for introducing algebraic techniques in his treatise *Hisab al-jabr w'al-muqabala* (the word "algebra" derived from "al-jabr" and the word "algorithm" from his name). Other celebrated Islamic mathematicians at the House included al-Kindi and al-Hajjaj.

Clagett notes that the *Verba filiorum* is important for Western geometry both because "it gave the formulas for the area of the circle, the area and volumes of a sphere, and so on . . . and it also presented demonstrations of an Archimedean character of these formulas" (223) for the first time in the West.

The contributions of the treatise according to Clagett are:

1) An exhaustion proof of Prop. 1 of *Measurement of the Circle (MC)* based on Archimedes' proof
2) Calculation of π "drawn from" Archimedes' *MC* Prop. 3 and Eutocius' commentary
3) Hero's theorem for the area of a triangle in terms of its sides
4) Theorems (and geometric proofs) for the surface area and volume of a cone
5) Theorems for the surface area and volume of a sphere with "demonstrations of an Archimedean character"
6) Use of a formula equivalent to πr^2 for the area of a circle, not just Archimedes' cr/2
7) The 1st example in "the West of the problem of finding two mean proportionals between two given quantities," with two solutions (those attributed to Archytas and Plato)
8) The 1st Latin solution of trisecting an angle
9) "A method of approximating cube roots to any desired limit."

This tract helped the spread of Archimedean geometry (both results and some of its methods of proof). For example, Roger Bacon mentions it in a list of works on geometry (13th century), and Leonardo Fibonacci "borrows heavily" from *Verba filiorum* in his 1220 revised *Practica geometrie*, as did his contemporary Jordanus de Nemore. (224) In the 14th century, Thomas Bradwardine (or Pseudo-Bradwardine) appeared to be "aware of Proposition IV from the *Verba filiorum*" (225).