Name:

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TTH 11:30 Philip Engel
TTH 11:30 Alison Miller

- Start by writing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly and except for problems 1-3, give details. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
14		10
Total:		150

Problem 1) (20 points) True or False? No justifications are needed.

1) T F The matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ has at least one real positive eigenvalue.

Solution:
It can already be seen from the fact that the trace is positive.

2) T F Every 3×2 matrix has a non-zero vector in its kernel.

Solution:
For 2×3 it would have been true.

3) T F For any 3×3 matrix A satisfying $A^2 = 0$, the image of A is a subspace of the kernel of A.

Solution:
Yes, if y is in the image, then $y = Ax$, but then $Ay = A^2y = 0$ so that y is also in the kernel.

4) T F If an invertible 2×2 matrix A is diagonalizable then the matrix A^{-2} is diagonalizable.

Solution:
$A = S^{-1}BS$ implies $A^{-2} = (S^T)^{-1}B^{-2}S^T$.

5) T F The Fourier series of the function $f(x) = \sin(5x)$ is $\sin(5x)$.

Solution:
The function is already a Fourier series.i It has only one term.

6) T F The space of smooth functions satisfying $f(x) = \sin(x)f(-x)$ forms a linear space.

Solution:
Check the three conditions: closed under addition, scaling multiplication and 0.

7) T F The geometric multiplicity of an eigenvalue 0 of A is equal to the nullity of A.

Solution:
The geometric multiplicity is the nullity.

8) T F Let A, B be arbitrary 3×3 matrices. The eigenvalues of $A + B$ are the sum of the eigenvalues of A and B.

Solution:
Take a matrix A and then $-A$.

9) T F A continuous dynamical system $x'(t) = Ax(t)$ with a 2×2 matrix A is asymptotically stable if $\text{tr}(A) < 0$.

Solution:
A negative trace does not necessarily imply that all eigenvalues are negative. Take $A = Diag(2, -3)$ for example.

10) T F If $\vec{x}(t+1) = A\vec{x}(t)$ is an asymptotically stable dynamical system, then each real eigenvalue λ of A satisfies $\lambda < 0$.

Solution:
Yes, this assures it.

11) T F The sum of two reflection dilation matrices is a reflection dilation matrix.

Solution:
All reflection-dilation matrices are of the same type $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$.

12) T F The function $f(x,t) = \sin(3x)\cos(3t)$ solves the heat equation $f_t = f_{xx}$.

Solution:
It satisfies the wave equation, not the heat equation.

13) T F If a system of linear equations $A\vec{x} = \vec{c}$ with 2×2 matrix A has 2 different solutions, there exists $\vec{b}$ such that $A\vec{x} = \vec{b}$ has no solution.

Solution:
There is a nontrivial kernel. The image is therefore not the entire space.

14) T F The equilibrium point $(0, 0)$ of the nonlinear system $x' = -2x$, $y' = -3y^3 - y$ is asymptotically stable.

Solution:
It is not stable because the eigenvalues are zero.

15) T F All projection matrices are diagonalizable.

Solution:
They are symmetric.

16) T F Using the length that comes from the Fourier series inner product, $||3\sin(5x) - 4\sin(10x)|| = 5$.

Solution:
This is a consequence of the Parseval equality.

17) T F The product of the eigenvalues of a symmetric non-invertible 2×2 matrix is 0.

Solution:
The product of the eigenvalues is the determinant

18) T F All solutions to the differential equation $x''(t) + 9x(t) = \sin(9t)$ stay bounded.

Solution:
This is not a resonance case.

19) T F For any rotation matrix A, the transpose A^T is similar to A.

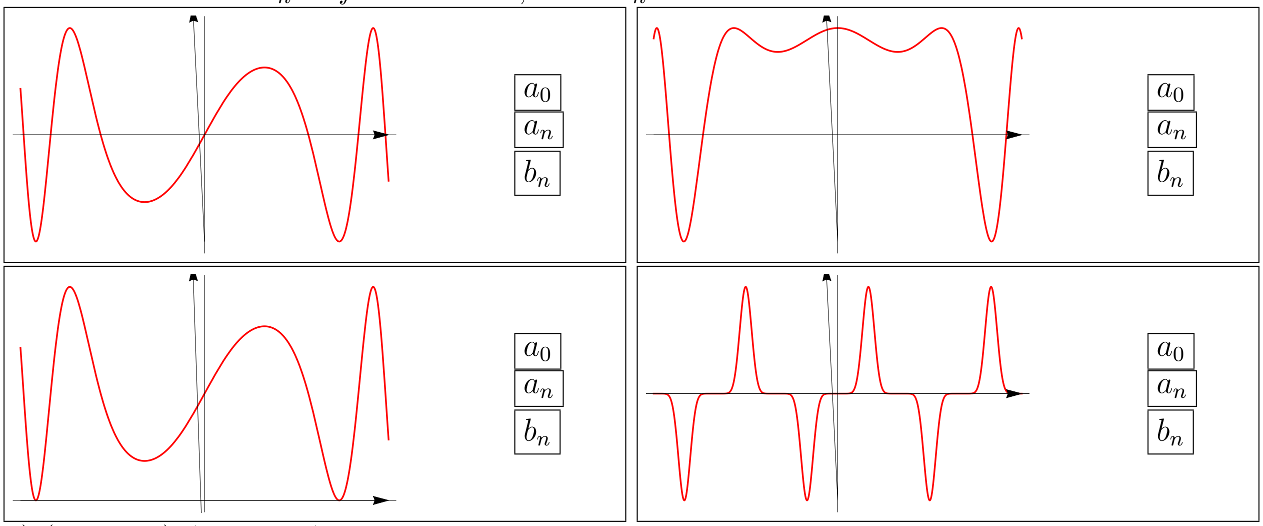
Solution:
This was probably the hardest TF problem in this exam. We expected you to see this in two or three dimensions and then get the intuition that the transpose is the inverse which is again a rotation of the same type (watch the rotation time ,reversed). Also possible is of course to "know" that A and A^T are similar in full generality but to see this, one needs a theorem called Jordan normal form theorem. For rotations, one can also see it by diagonalization over the complex numbers. Both matrices A and A^T are similar to the same diagonal matrix because A and A^T have the same eigenvalues. So, there exists a complex S for which $SA = A^T S$. Because also $\overline{S}A = A^T\overline{S}$, we can sum up the equations and get $(S+\overline{S})A = A^T(S+\overline{S})$ showing that A and A^T are conjugated by a real coordinate change.

20) | T | **F** | If the sum of the squares of the entries of a square matrix is 1, then A is invertible.

Solution:
Just take a counter example like the diagonal matrix $\mathrm{Diag}(1, 0)$.

Problem 2) (10 points) No justifications needed

a) (3 points) The numbers a_n, b_n are Fourier coefficients. If the coefficient of f with respect to the constant function is zero, circle a_0, if the cos-coefficients a_n of f are all zero, circle a_n, if the sin-coefficients b_n of f are all zero, circle b_n.



b) (3 points) Assume A is a $n \times n$ matrix for which

$$A^2 - A = 0 .$$

Check what is always true. We write O_n for the 0 matrix.

Solution:
a) a_0, a_n are checked.
b) b_n is checked.
c) a_n is checked.
d) a_0, a_n is checked.

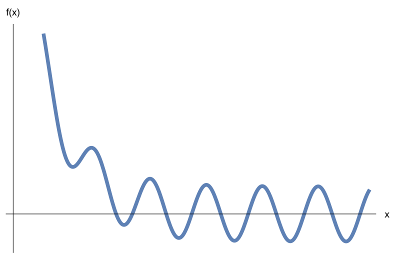
Statement	Always true
$A = 0_n$ or $A = I_n$	
A is not invertible	
At least one of the eigenvalue is 0 or 1	
A is a reflection	
A is orthogonal	
The trace of A is zero	

Solution:
At least one of the eigenvalues is 0 or 1 is true.

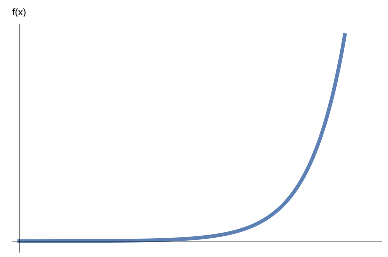
c) (4 points) Match the differential equations with possible solution graphs. There is an exact match.

Enter A-D	Differential equation
	$f'(t) - 16f(t) = 2\sin(4t)$
	$f''(t) + 16f(t) = 2\sin(16t)$

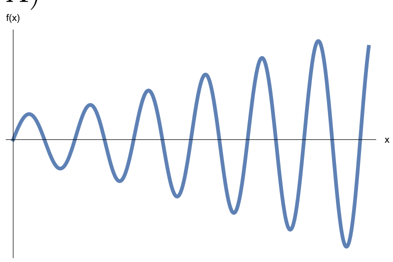
Enter A-D	Differential equation
	$f'(t) + 16f(t) = 2\sin(4t)$
	$f''(t) + 16f(t) = 2\sin(4t)$



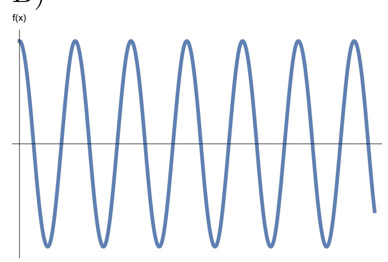
A)



B)



C)



D)

Solution:
B, A
D, C

Problem 3) (10 points) No justifications needed

a) (4 points) Assume T is a transformation on C^∞_{per}, the linear space of 2π-periodic functions on the real line. Which T are linear?

Transformation	Check if linear
$Tf(x) = f(x+1)$	
$Tf(x) = f(\cos(x))$	
$Tf(x) = f'(x + \cos(x))$	

Transformation	Check if linear
$Tf(x) = f(f(x)\cos(x))$	
$Tf(x) = \cos(f(x))$	
$Tf(x) = \cos(x) + f(x)$	

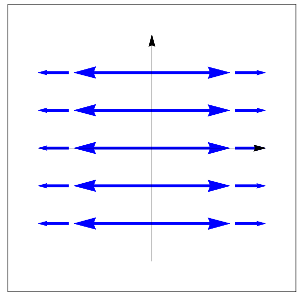
Solution:
Three three on the left hand side are linear, the three on the right not.

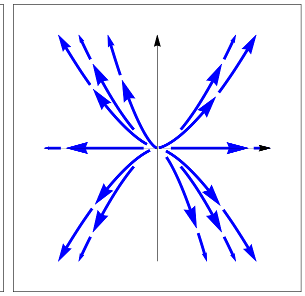
b) (4 points) Match the differential equation

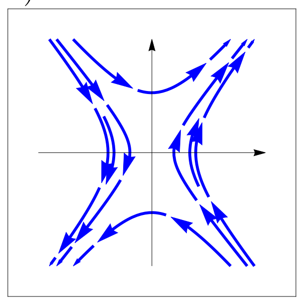
$$\frac{d}{dt}\vec{x}(t) = A\vec{x}(t)$$

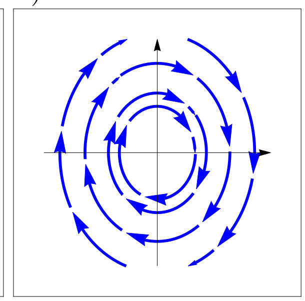
with the phase portraits. There is an exact match.

Matrix	Enter a) - d)
$A = \begin{bmatrix} 0 & 2 \\ -3 & 0 \end{bmatrix}$	
$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$	
$A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$	
$A = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$	


a)

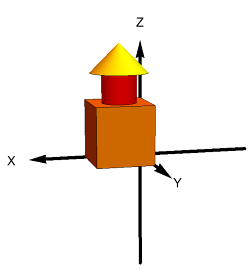

b)


c)

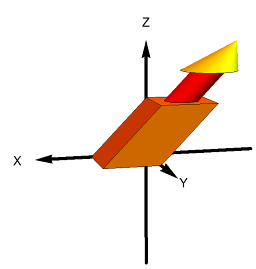

d)

Solution:
D,B,C,A

c) (2 points) The following transformation T has been implemented by one of the matrices A,B,C or D. Which one?



is mapped by T to

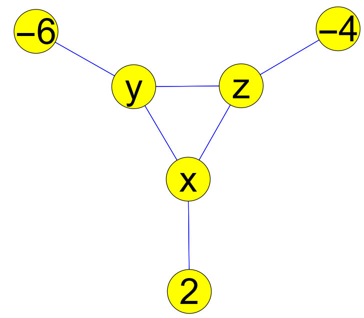


A= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ B= $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C= $\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ D= $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution:
B) Like all these type of problems, you have to look at the colums of the matrix.

Problem 4) (10 points)

We look at a **social network**, where the values at nodes are opinions which can be both positive or negative. Some members have fixed opinions already and are not influenced by their friends. The rule is that the remaining people have the sum of the opinions of their friends. We still miss the opinions of three people x, y, z.



$$\left| \begin{array}{rcrcrl} x & - & y & - & z & = 2 \\ -x & + & y & - & z & = -6 \\ -x & - & y & + & z & = -4 \end{array} \right|$$

a) (7 points) Write down the augmented 3×4 matrix $B = [A|b]$ which belongs to the system and find its row reduced echelon form. We ask you here to document which row reduction steps you do.

b) (3 points) What are the solutions to the system of linear equations?

Solution:
The augmented matrix of the system is

$$B = [A|b] = \begin{bmatrix} 1 & -1 & -1 & 2 \\ -1 & 1 & -1 & -6 \\ -1 & -1 & 1 & -4 \end{bmatrix} .$$

Row reduced, this is

$$\text{rref}(B) = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} .$$

b) The solution can be read off in the last column. It is $x = 5, y = 1, z = 2$.

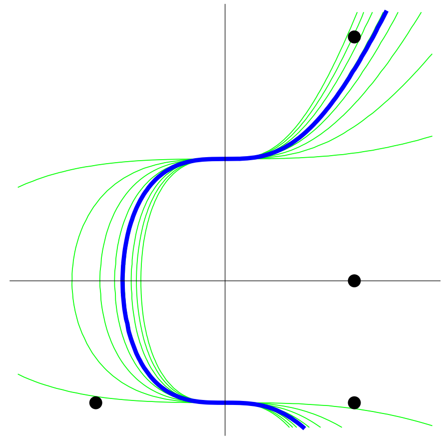
Problem 5) (10 points)

Using the least square method, find the best function

$$a + bx^3 = y^2$$

which best fits the data points (x, y):

$$\{(1,0), (1,2), (-1,-1), (1,-1)\ \}\ .$$



Solution:
As usual, we write down the equations:

$$\begin{array}{rcl} a+b &=& 0 \\ a+b &=& 4 \\ a-b &=& 1 \\ a-b &=& 1\ . \end{array}$$

This is a system of equations $A\vec{v} = \vec{b}$ which leads to the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$$

and the vector $\vec{b} = \begin{bmatrix} 0 \\ 4 \\ 1 \\ 1 \end{bmatrix}$. We get the solution with $\vec{x} = (A^T A)^{-1} A^T \vec{b}$. We have $A^T A)^{-1} = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}/12$ and $A^T\vec{b} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$. Now $\vec{x} = \begin{bmatrix} 4/3 \\ 1/3 \end{bmatrix}$. The best solution is $4/3 + x^3/3 = y^2$.

Problem 6) (10 points)

a) (6 points) Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

Find a matrix S and a diagonal matrix B such that $S^{-1}AS = B$.

b) (4 points) Find the closed form solution of the discrete dynamical system

$$\vec{v}(t+1) = A\vec{v}(t)$$

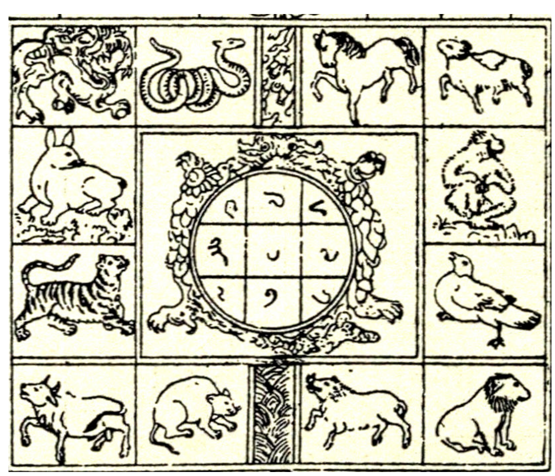
with initial condition $\vec{v}(0) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

Solution:
a) The matrix B is a diagonal matrix which contains the eigenvalues. It is $\mathrm{Diag}(0,0,0,30)$. We have got the last eigenvalue from the trace. The matrix S contains the eigenvectors as columns:

$$\begin{bmatrix} -2 & -3 & -4 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}.$$

b) The initial condition is an eigenvector. We have $v(t) = 30^t \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$.

Problem 7) (10 points)



According to the legend of **Lo Shu**, there had been a huge flood around 650 BC. As **king Yu** tried to channel the water to the sea, a turtle emerged with a 3×3 grid pattern on its shell. The numbers in each row, column and diagonal added up to 15, the number of days in each of the 24 cycles of the Chinese solar year. The pattern helped the king to control the river. The **Lo Shu square** is an example of a "magic square": $A = \begin{bmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{bmatrix}$.

a) (2 points) What is the sum of the eigenvalues of A?

b) (2 points) Find the eigenvalue of A to the eigenvector $[1,1,1]^T$.

c) (2 points) What is the determinant of A?

d) (2 points) The matrix A has two more eigenvalues $\pm i\sqrt{k}$, where k is an integer. Find k.

e) (2 points) The matrix $B = A/15$ is an example of a **double stochastic matrix**, a matrix for which all rows and columns add to 1. What are the eigenvalues of B?

Solution:
a) The trace is 15.
b) The maximal eigenvalue is 15.
c) The determinant is 360.
d) Since the determinant is the product of the eigenvalues we have $k = 24$.
e) The eigenvalues are $1, \pm i\sqrt{24}/15$.

Problem 8) (10 points)

In the Mathematica project we have looked at **Kirchhoff matrices**. Here is that matrix for the cyclic graph C_7:

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} .$$

a) (3 points) You see that $[1, 1, 1, 1, 1, 1, 1]^T$ is an eigenvector. What is the eigenvalue?

b) (3 points) Find the eigenvalues of A.

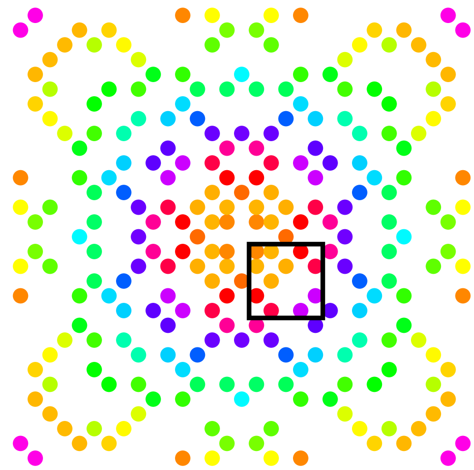
c) (4 points) Write down the eigenvectors of A.

Solution:
a) Just plug in. The eigenvalue is 0.
b) Write $A = -Q - Q^T + 2$. The eigenvalues are $2 - e^{2\pi ik/7} - e^{-2\Pi ik/7}$, where $k = 0, 1, 2, 3, 4, 5, 6$.
c) The eigenvectors are the same than the ones for Q: $[1, \lambda_k, \lambda_k^2, \dots, \lambda_k^6]$.

Problem 9) (10 points)

a) (4 points) Evaluate the determinant of

$$A = \begin{bmatrix} 18 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 7 & 17 & 7 & 7 & 7 & 7 & 7 & 7 \\ 6 & 6 & 16 & 6 & 6 & 6 & 6 & 6 \\ 5 & 5 & 5 & 15 & 5 & 5 & 5 & 5 \\ 4 & 4 & 4 & 4 & 14 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 & 13 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 12 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 11 \end{bmatrix}$$



In this problem we worked with the matrix A_5. The **21b conjecture** is: all matrices A_n are invertible for $n > 28$.

b) (3 points) The matrix A is defined by **Gaussian primes** $A(n,m) = 1$ if $n - im$ is prime and $A(n,m) = 0$ if not. You don't need to know about Gaussian primes to find the determinant of

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

c) (3 points) Evaluate the determinant of the matrix

$$A = \begin{bmatrix} 8 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} .$$

Solution:
a) $10^7 46$.
b) 1
c) 5040.

Problem 10) (10 points)

Solve the following differential equations:

a) (2 points)

$$f'''(t) = e^t$$

b) (2 points)

$$f'(t) + f(t) = e^t$$

c) (2 points)

$$f'(t) - f(t) = e^t$$

d) (2 points)

$$f''(t) - f(t) = e^t$$

e) (2 points)

$$f''(t) + f(t) = e^t$$

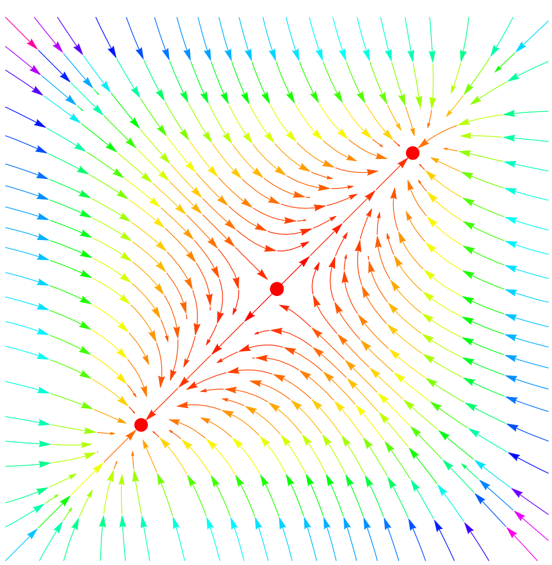
Solution:
a) $e^t + C_1 t^2 + C_2 t + C_3$.
b) $C_1 e^{-t} + e^t/2$
c) $C_1 e^t + te^t$
d) $C_1 e^t + c_2 e^{-t} + te^t/2$
e) $C_1 \cos(t) + c_2 \sin(t) + e^t/2$.

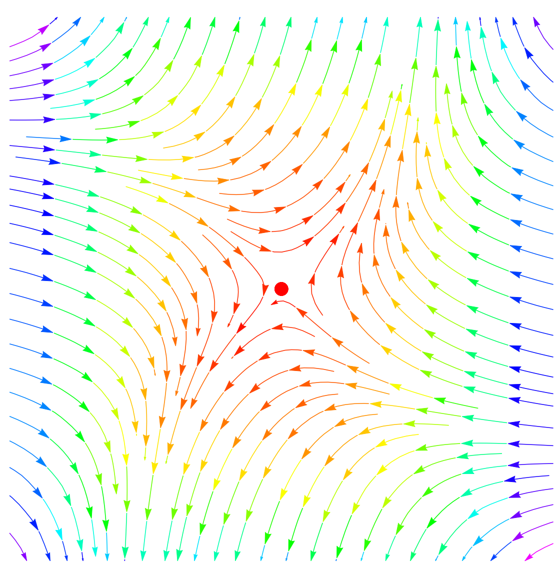
Problem 11) (10 points)

Find and analyze the stability of the equilibrium points of the nonlinear dynamical system

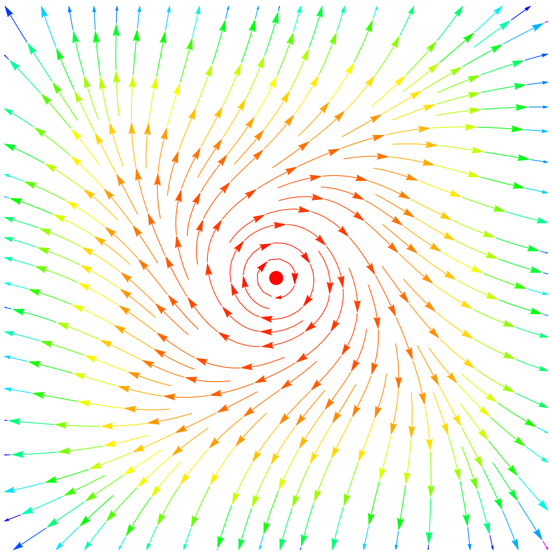
$$\begin{aligned} \frac{d}{dt}x &= y - x^3 \\ \frac{d}{dt}y &= x - y^3 . \end{aligned}$$

As usual, we ask you to find the **null clines**, the **equilibrium points** as well want you to perform a **linear stability analysis** at those points using the Jacobian. Which of the following four phase portraits fits the system?

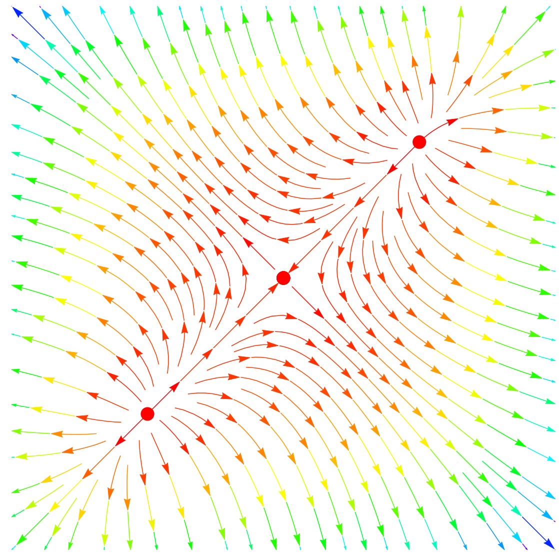




A



B



C

D

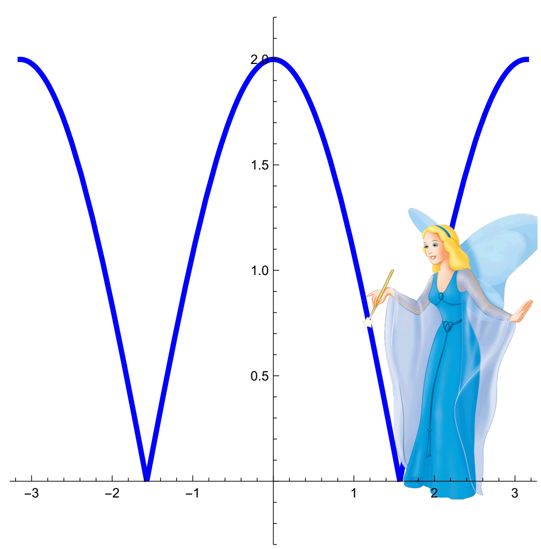
Solution:
The nullclines are $y = x^3$ and $x = y^3$. The equilibria are $(0,0),(1,1),(-1,-1)$. The Jacobean is $\begin{bmatrix} -3x^2 & 1 \\ 1 & -3y^2 \end{bmatrix}$. At $(0,0)$¡ the eigenvalues are ± 1, which is not stable. At $(1,1)$ the eigenvalues are -3 ± 1 which is stable. at $(-1,-1)$, the eigenvalues are -3 ± 1 again, which is stable. The right phase portrait is A).

Problem 12) (10 points)

a) (6 points) Find the **Fourier series** of the function

$$f(x) = 2|\cos(x)| \; .$$

The graph of the function f on $[-\pi, \pi]$ is displayed to the right.



Hint: A **blue fairy** gives you the identities

$$2\cos(x)\cos(nx) = \cos((n+1)x) + \cos((n-1)x)$$

$$2\cos(x)\sin(nx) = \sin((n+1)x) + \sin((n-1)x)$$

b) (4 points) Find the value of the sum of the squares of all the Fourier coefficients of f.

Solution:
a) The function is even so that there is a cosine series. We have $a_0 = 4\sqrt{2}/\pi$ and $a_n = (4/\pi)(\sin((n+1)\pi/2)/(n+1) + \sin((n-1)\pi/2)/(n-1))$ (you did not have to worry about the case $n = 1$, it is zero. One can see this by direct computation in the case n=1). The Fourier series is $a_0/\sqrt{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$.
b) Use Parseval. The sum is $||f||^2 = \frac{2}{\pi}\int_0^{\pi} 4\cos^2(x)\,dx = 4$.

Problem 13) (10 points)

a) (5 points) Solve the **modified heat equation**

$$u_t = 7u_{xx} + u_{xxxxxx}$$

with initial condition $u(x, 0) = \sin(11x) + 5\sin(22x)$.

b) (5 points) Solve the **modified wave equation**

$$u_{tt} = 7u_{xx} + u_{xxxxxx}$$

with initial condition $u(x, 0) = e^x$ and $u_t(x, 0) = 0$.

Hint: The **oracle of Delphi** gives you the identities

$$\int_0^{\pi} e^x \sin(nx)\,dx = n(1 - e^{\pi}(-1)^n)/(1 + n^2)$$

$$\int_0^{\pi} e^x \cos(nx)\,dx = (e^{\pi}(-1)^n - 1)/(1 + n^2)$$

Solution:
a) $\sin(11x)e^{(-11^6-7\cdot 11^2)t} + 5\sin(22x)e^{(-22^6-7\cdot 22^2)t}$.
b) $b_n = (2/\pi)\frac{n(1-e^{\pi}(-1)^n}{n^2+1}$. We have

$$u = \sum_n b_n \sin(nx)\cos(\sqrt{n^6 + 7n^2}t) .$$

Problem 14) (10 points)

We love determinants. Can not get enough of them. Anyway, here is an other one for a bonus 14 problem (the actual exam always has 13 problems as you might have noticed [(1)] ...). Find the determinant of

$$A = \begin{bmatrix} 9 & 1 & 1 & 1 & 1 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 1 & 9 & 1 & 1 & 1 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 1 & 1 & 9 & 1 & 1 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 1 & 1 & 1 & 9 & 1 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 1 & 1 & 1 & 1 & 9 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 8 & 8 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 & 1 \end{bmatrix} .$$

(1) Actually, some students in the past have been even done statistics (big data!) like on what the expected number of True answers are in the TF-section of Math21b. As in average in the last dozen practice tests, there used to be 11.8 True from 20 problems, a strategy was to just mark everything True, scoring at least 12 points in that section. But we might (for fun) once design a TF problem 1), where all 20 problems are False ... (and then maybe have one True), but still get that 11.8 average down a bit. Anyway, instead of "big data" it can be a better strategy to understand as many TF problems as possible.

Solution:
It is a partitioned matrix. Each block can be dealt with in a different way. For the first block, subtract 8 from the diagonal to get the eigenvalues 0,0,0,0,5+8,8,8,8,8,8. Which gives the determinant $8^4 * 13 = 458752$. The second block has only one pattern $-3 * 1 * 5 * 4 * 2 = -120$ (negative because we have 5 up-crossings. The third block can be row reduced to have determinant 1. The product $8^4 * 13 * (-120)$ is $\boxed{-6389760}$.