

We find the QR decomposition of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

by doing Gram-Schmidt orthogonalization of its column vectors \vec{v}_i .

$$\vec{u}_1 = \vec{v}_1 = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \qquad \vec{w}_1 = \vec{u}_1 / \|\vec{u}_1\| = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix}$$

$$\vec{u}_2 = \vec{v}_2 - (\vec{v}_2 \cdot \vec{w}_1) \vec{w}_1 = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \qquad \vec{w}_2 = \vec{u}_2 / \|\vec{u}_2\| = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix}$$

$$\vec{u}_3 = \vec{v}_3 - (\vec{v}_3 \cdot \vec{w}_1) \vec{w}_1 - (\vec{v}_3 \cdot \vec{w}_2) \vec{w}_2 = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \qquad \vec{w}_3 = \vec{u}_3 / \|\vec{u}_3\| = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix}$$

$$Q = \begin{bmatrix} | & | & | \\ \vec{w}_1 & \vec{w}_2 & \vec{w}_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix}$$

$$R = \begin{bmatrix} \|\vec{u}_1\| & \vec{w}_1 \cdot \vec{v}_2 & \vec{w}_1 \cdot \vec{v}_3 \\ 0 & \|\vec{u}_2\| & \vec{w}_2 \cdot \vec{v}_3 \\ 0 & 0 & \|\vec{u}_3\| \end{bmatrix} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix}.$$