

1) $A = \begin{bmatrix} 4 & -1 \\ 3 & 0 \end{bmatrix}$

Answer: Technique: Eigenvalues $\lambda = 1, 3$ with eigenvectors $[1, 1]^T, [1, 3]^T$.

2) Find all eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 3 \\ 3 & 1 & 1 & 2 \\ 1 & 3 & 2 & -1 \end{bmatrix}.$$

Answer: The eigenvalue 1 can be seen using the eigenvector $(1, 1, 1, 1)$. You might also see the eigenvector $(0, 1, 1, 0)$ to the eigenvalue 5. Then there are the eigenvalues $-5, 3$.

3) Find all the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}.$$

Answer: $A + 1$ has eigenvalues 8, 0, 0, 0 so that A has eigenvalues 7, $-1, -1, -1$.