Review Problems

1. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$, and let $\vec{b} \in \mathbb{R}^2$. What can you say about the number of solutions of the system $A\vec{x} = \vec{b}$?

2. Let L be the line $y = 3x$ in $\mathbb{R}^2$. We want to find the matrix of proj_L using coordinates.

   (a) Find a basis $\mathfrak{B}$ consisting of vectors whose projection is easy to compute.

   (b) Find the $\mathfrak{B}$-matrix of proj_L.

   (c) Find the standard matrix of proj_L.

3. Find a matrix A such that $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of A with eigenvalue 2 and $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is an eigenvector of A with eigenvalue 3.

4. True or false: If A is an invertible $n \times n$ matrix and $\vec{v}, \vec{w}$ are orthogonal vectors in $\mathbb{R}^n$, then $A^T \vec{v}$ is orthogonal to $A^{-1}\vec{w}$.

5. If $A^T A$ is a diagonal matrix, what can you say about the columns of A?

6. (§7.5, #19) Let V be a subspace of $\mathbb{R}^n$ with $\dim(V) = m < n$.

   (a) If A is the matrix of proj_V, find the trace and determinant of A.

   (b) If B is the matrix of ref_V, find the trace and determinant of B.

7. (a) Let A be a symmetric 3×3 matrix such that $A^2 = I_3$. What does A do geometrically?

   (b) Let B be a symmetric 3×3 matrix such that $B^2 = B$. What does B do geometrically?

8. (a) Find all matrices in Jordan normal form which have characteristic polynomial $-\lambda^3$.

   (b) True or false: If A has characteristic polynomial $-\lambda^3$, then A^3 is the zero matrix.