SECOND PRACTICE EXAM SECOND HOURLY **Math 21a,Spring 2003**

Name:

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

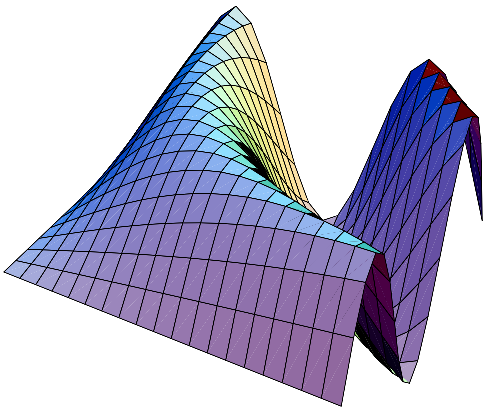
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) Circle the correct letter. No justifications are needed. The score for this question is the number of correct answers.

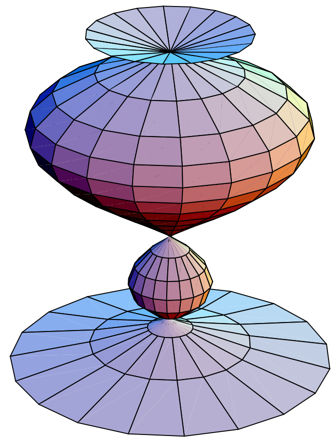
T	F	A function $f(x,y)$ on the plane for which the absolute minimum and the absolute maximum are the same must be constant.
T	F	The functions $f(x,y)$ and $g(x,y) = f(x,y) + 2002$ do not have the same critical points.
T	F	The sign of the Lagrange multiplier tells whether the critical point of $f(x,y)$ constrained to $g(x,y) = 0$ is a local maximum or a local minimum.
T	F	The gradient of a function $f(x,y,z)$ is tangent to the level surfaces of f
T	F	The point $(0,1)$ is a local minimum of the function $x^3+(\sin(y-1))^2$.
T	F	For any curve, the acceleration vector $r''(t)$ of $r(t)$ is orthogonal to the velocity vector at $r(t)$.
T	F	If $D_u f(x,y,z) = 0$ for all unit vectors u, then (x,y,z) is a critical point.
T	F	$\int_a^b \int_c^d x \; dxdy = (d^2 - c^2)(b-a)/2$, where a,b,c,d are constants.
T	F	The functions $f(x,y)$ and $g(x,y) = (f(x,y))^2$ have the same critical points.
T	F	If a function $f(x,y) = ax + by$ has a critical point, then $f(x,y) = 0$ for all (x,y).
T	F	$f_{xyxyx} = f_{yyxxx}$ for $f(x,y) = \sin(\cos(y + x^{14}) + \cos(x))$.
T	F	The function $f(x,y) = -x^{2002} - y^{2002}$ has a critical point at $(0,0)$ which is a local minimum.
T	F	It is possible that for some unit vector u, the directional derivative $D_u f(x,y)$ is zero even though the gradient $\nabla f(x,y)$ is nonzero.
T	F	If (x_0,y_0) is the maximum of $f(x,y)$ on the disc $x^2+y^2 \leq 1$ then $x_0^2 + y_0^2 < 1$.
T	F	The linear approximation $L(x,y,z)$ of the function $f(x,y,z) = 3x+5y-7z$ at $(0,0,0)$ satisfies $L(x,y,z) = f(x,y,z)$.
T	F	If $f(x,y) = \sin(x) + \sin(y)$, then $-\sqrt{2} \leq D_u f(x,y) \leq \sqrt{2}$.
T	F	There are no functions $f(x,y)$ for which every point on the unit circle is a critical point.
T	F	An absolute maximum (x_0,y_0) of $f(x,y)$ is also an absolute maximum of $f(x,y)$ constrained to a curve $g(x,y) = c$ that goes through the point (x_0,y_0).
T	F	If $f(x,y)$ has two local maxima on the plane, then f must have a local minimum on the plane.
T	F	$\int\int_D f(x,y)g(x,y) \; dA = (\int\int_D f(x,y) \; dA)(\int\int_D g(x,y) \; dA)$ is true for all functions f and g.

Problem 2) (10 points)

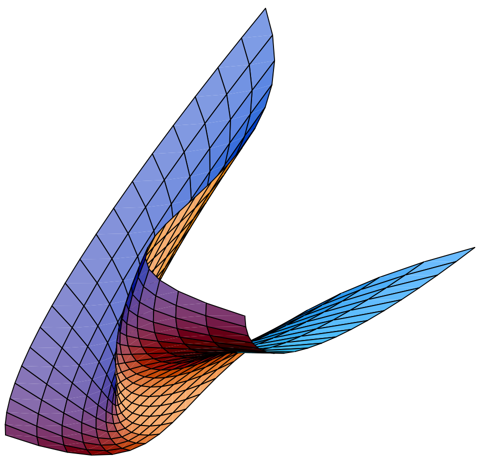
Match the parametric surfaces with their parameterization. No justification is needed.



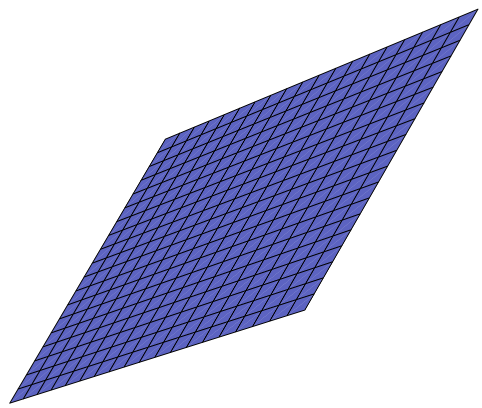
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$(u, v) \mapsto (u, v, u + v)$
	$(u, v) \mapsto (u, v, \sin(uv))$
	$(u, v) \mapsto (0.2 + u(1 - u^2)) \cos(v), (0.2 + u(1 - u^2)) \sin(v), u)$
	$(u, v) \mapsto (u^3, (u - v)^2, v)$

Problem 3) (10 points)

Match the integrals with those obtained by changing the order of integration. No justifications are needed.

Enter I,II,III,IV or V here.	Integral
	$\int_0^1 \int_{1-y}^1 f(x,y)\ dxdy$
	$\int_0^1 \int_y^1 f(x,y)\ dxdy$
	$\int_0^1 \int_0^{1-y} f(x,y)\ dxdy$
	$\int_0^1 \int_0^y f(x,y)\ dxdy$

I) $\int_0^1 \int_0^x f(x,y)\ dydx$

II) $\int_0^1 \int_0^{1-x} f(x,y)\ dydx$

III) $\int_0^1 \int_x^1 f(x,y)\ dydx$

IV) $\int_0^1 \int_0^{x-1} f(x,y)\ dydx$

V) $\int_0^1 \int_{1-x}^1 f(x,y)\ dydx$

Problem 4) (10 points)

Consider the graph of the function $h(x,y) = e^{-3x-y} + 4$.

1. Find a function $g(x,y,z)$ of three variables such that this surface is the level set of g.
2. Find a vector normal to the tangent plane of this surface at (x,y,z).
3. Is this tangent plane ever horizontal? Why or why not?
4. Give an equation for the tangent plane at $(0,0)$.

Problem 5) (10 points)

Find all the critical points of the function $f(x,y) = \frac{x^2}{2} + \frac{3y^2}{2} - xy^3$. For each, specify if it is a local maximum, a local minimum or a saddle point and briefly show how you know.

Problem 6) (10 points)

Minimize the function $E(x,y,z) = \frac{k^2}{8m}(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2})$ under the constraint $xyz = 8$, where k^2 and m are constants.

Remark. In quantum mechanics, E is the ground state energy of a particle in a box with dimensions x, y, z. The constant k is usually denoted by $\hbar$ and called the Planck constant.

Problem 7) (10 points)

Assume $F(x,y) = g(x^2 + y^2)$, where g is a function of one variable. Find $F_{xx}(1,2) + F_{yy}(1,2)$, given that $g'(5) = 3$ and $g''(5) = 7$.

Problem 8) (10 points)

Consider the region inside $x^2 + y^2 + z^2 = 2$ above the surface $z = x^2 + y^2$.

a) Sketch the region.

b) Find its volume.

Problem 9) (10 points)

Draw the gradient vector field of $f(x,y) = xy - 2x$ together with a contour map of f.