Name:

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- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

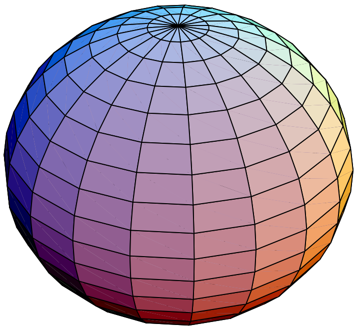
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) Circle the correct letter. No justifications are needed. The score for this question is the number of correct answers

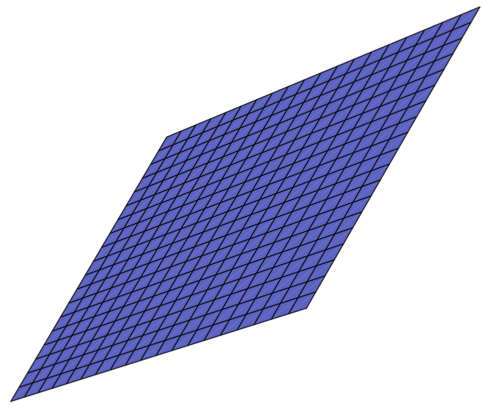
T	F	At a local maximum (x_0, y_0) of $f(x, y)$, one has $f_{yy}(x_0, y_0) \geq 0$.
T	F	If R is the region bounded by $x^2 + 4y^2 = 1$ then $\int\int_R xy^4 \, dxdy < 0$.
T	F	The gradient $\langle 2x, 2y \rangle$ is perpendicular to the surface $z = x^2 + y^2$.
T	F	The equation $f(x, y) = k$ implicitly defines x as a function of y and $\frac{dx}{dy} = \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}$.
T	F	$f(x, y) = \sqrt{(16 - x^2 - y^2)}$ has both an absolute maximum and an absolute minimum on its domain of definition.
T	F	If (x_0, y_0) is a critical point of $f(x, y)$ under the constraint $g(x, y) = 0$, and $f_{xy}(x_0, y_0) < 0$, then (x_0, y_0) is a saddle point.
T	F	The vector $r_u(u, v)$ of a parameterized surface $(u, v) \mapsto r(u, v) = (x(u, v), y(u, v), z(u, v))$ is normal to the surface.
T	F	The identity $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) \, dydx = \int_0^1 \int_0^{\pi/2} r^2 \, d\theta dr$ holds.
T	F	$f(x, y)$ and $g(x, y) = f(x^2, y^2)$ have the same critical points.
T	F	If $f(x, t)$ satisfies the Laplace equation $f_{xx} + f_{tt} = 0$ and simultaneously the wave equation $f_{xx} = f_{tt}$, then $f(x, t) = ax + bt + c$.
T	F	Every smooth function satisfies the partial differential equation $f_{xxyy} = f_{xyxy}$.
T	F	The function $(x^4 - y^4)$ has has neither a local maximum nor a local minimum at $(0, 0)$.
T	F	$\int_0^1 \int_0^{\pi/2} r \, d\theta dr = \pi/4$.
T	F	At a saddle point, the directional derivative is zero for two different vectors u, v.
T	F	It is possible to find a function of two variables which has no maximum and no minimum.
T	F	The value of the function $f(x, y) = e^x y$ at $(0.001, -0.001)$ can by linear approximation be estimated as -0.001.
T	F	For any function $f(x, y, z)$ and any unit vectors u, v, one has the identity $D_{u \times v} f(x, y, z) = D_u f(x, y, z) D_v f(x, y, z)$.
T	F	Given 2 arbitrary points in the plane, there is a function $f(x, y)$ which has these points as critical points and no other critical points.
T	F	The maximum of $f(x, y)$ under the constraint $g(x, y) = 0$ is the same as the maximum of $g(x, y)$ under the constraint $f(x, y) = 0$.
T	F	Assume (x_0, y_0) is a critical point of $f(x, y)$ and $f_{xx} f_{yy} - f_{xy}^2 \neq 0$ at this point. Let T be the tangent plane of the surface $S = \{f(x, y) - z = 0\}$ at $P = (x_0, y_0, f(x_0, y_0))$. If the intersection of T with S is a single point, then (x_0, y_0) is a local max or local min.

Problem 2) (10 points)

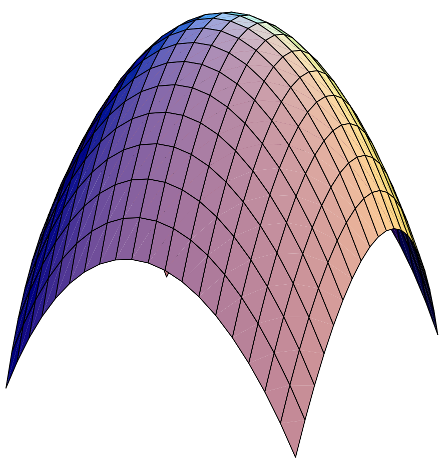
Match the parametric surfaces $S = \vec{r}(R)$ with the corresponding surface integral $\int\int_S \; dS = \int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv$. No justifications are needed.



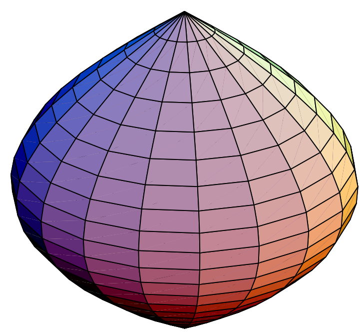
I



II



III



IV

Enter I,II,III,IV here	Surface integral
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^1 \int_0^1 \sqrt{1+4u^2+4v^2} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^1 \int_0^1 \sqrt{3} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^{2\pi} \int_0^{\pi} \sin(v)\sqrt{1+\cos(v)^2} \; dvdu$
	$\int\int_R |\vec{r}_u \times \vec{r}_v| \; dudv = \int_0^{2\pi} \int_0^{\pi} \sin(v) \; dvdu$

Problem 3) (10 points)

Find all the critical points of the function $f(x,y) = xy(4 - x^2 - y^2)$. Are they maxima, minima or saddle points?

Problem 4) (10 points)

Let $f(x,y) = e^{(x-y)}$ so that $f(\log(2), \log(2)) = 1$. Find the equation for the tangent plane to the graph of f at $(\log(2), \log(2))$ and use it to estimate $f(\log(2) + 0.1, \log(2) + 0.04)$.

Problem 5) (10 points)

Find $\int\int\int_R z^2 \, dV$, where R of the solid obtained by intersecting $\{1 \leq x^2 + y^2 + z^2 \leq 4\}$ with the double cone $\{z^2 \geq x^2 + y^2\}$.

Problem 6) (10 points)

A **can** is a cylinder with a circular base. Its surface area (top, bottom and sides) is 300π cm^2. What is the maximum possible volume of such a can?

Problem 7) (10 points)

Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{xy^5}{x^2+y^2} \, dy \, dx$.

Problem 8) (10 points)

a) Find the area of the region D enclosed by the lines $x = \pm 2$ and the parabolas $y = 1 + x^2$, $y = -1 - x^2$.

b) Find the integral of $f(x,y) = y^2$ on the same region as in a). (The result can be interpreted as a moment of inertia).

Problem 9) (10 points)

Let $T(u,v) = (v\cos(u), 2v\sin(u)) = (x,y)$.

a) Find the image $S = T(R)$ of the rectangle $R = \{0 \leq u \leq \pi, 0 \leq v \leq 1\}$ under the map T and find its area using the formula for the change of variables.

b) Write the integral $\int_0^4 \int_{y/2}^{(y/2)+1} \frac{(2x+y)}{2} \, dxdy$ using uv coordinates with a change of variables $T(u,v) = (x,y) = (u+v, 2v)$ and evaluate that integral.