4/23/2003 SECOND HOURLY **Math 21a,Spring 2003**

Name:

MWF10 Ken Chung
MWF10 Weiyang Qiu
MWF11 Oliver Knill
TTh10 Mark Lucianovic
TTh11.5 Ciprian Manolescu

- Start by printing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points)

Circle the correct letter. No justifications are needed. Score:number of correct answers.

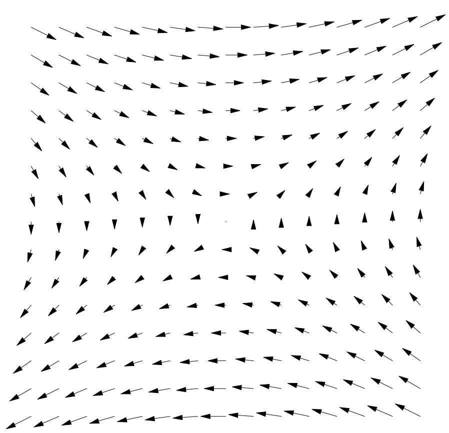
T	F	Given a function $f(x,y)$, then $F(x,y) = (f_x(x,y), f_x(x,y) + f_y(x,y))$ defines a vector field in the plane.
T	F	Every smooth function of three variables $f(x,y,z)$ satisfies the partial differential equation $f_{xyz} + f_{yzx} = 2f_{zxy}$.
T	F	If $f_x(x,y) = f_y(x,y)$ for all x, y, then $f(x,y)$ is a constant.
T	F	$(1,1)$ is a local maximum of the function $f(x,y) = x^2y - x + \cos(y)$.
T	F	If f is a smooth function of two variables, then the number of critical points of f inside the unit disc is finite.
T	F	The value of the function $f(x,y) = \sin(x+2y)$ at $(0.001, -0.002)$ can by linear approximation be estimated as -0.003.
T	F	A critical point for $f(x,y)$ on $\{(x-2)^2 + (y-2)^2 < 1\}$ is also a critical point for $g(x,y) = f(x^2, y^2)$.
T	F	There is no function $f(x,y,z)$ of three variables, for which every point on the unit sphere is a critical point.
T	F	If the double integral $\int\int_R f(x,y)\; dxdy$ is zero for a continuous function $f(x,y)$ and R is the unit disc, then $f(x,y) = 0$ for at least one point x, y.
T	F	The surface area is given by the formula $\int\int_R \vec{r}_u \times \vec{r}_v \; dudv$
T	F	The gradient of $f(x,y)$ is orthogonal to the level curves of f.
T	F	If (x_0, y_0) is a maximum of $f(x,y)$ under the constraint $g(x,y) = g(x_0,y_0)$, then (x_0,y_0) is a maximum of $g(x,y)$ under the constraint $f(x,y) = f(x_0,y_0)$.
T	F	The vector field $F(x,y) = (f_x(x,y), f_y(x,y))$ is orthogonal to the level curves of $f(x,y)$.
T	F	When changing to cylindrical coordinates, one has to include an integration factor $\rho^2 \sin(\phi)$.
T	F	The function $u(x,t) = x^2/2 + t$ solves the heat equation $u_t = u_{xx}$.
T	F	The vector $\vec{r}_u - \vec{r}_v$ is tangent to the surface parameterized by $\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$.
T	F	Fubini's theorem assures that $\int_0^1 \int_0^x f(x,y)\; dydx = \int_0^1 \int_0^y f(x,y)\; dxdy$.
T	F	If $(1,1,1)$ is a maximum of f under the constraints $g(x,y,z) = c, h(x,y,z) = d$, and the Lagrange multipliers satisfy $\lambda = 0, \mu = 0$, then $(1,1,1)$ is a critical point of f.
T	F	If $(0,0)$ is a critical point of $f(x,y)$ and the discriminant D vanishes but $f_{xx}(0,0) > 0$ then $(0,0)$ can not be a local maximum.
T	F	Let (x_0,y_0) be a saddle point of $f(x,y)$. For any unit vector $\vec{u}$ there are points arbitrarily close to (x_0,y_0) for which ∇f is parallel to $\vec{u}$.

Problem 2)	(10 points)

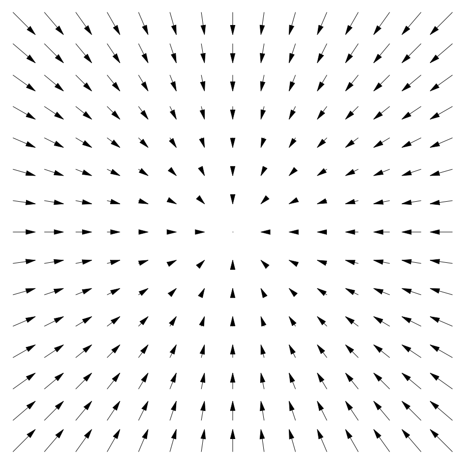
Circle the interior of the boxes to match the formulas of the vectorfields with the corresponding picture I,II,III or IV.

Vectorfield	I	II	III	IV
$F(x,y) = (2y, x)$				
$F(x,y) = (-x, -y)$				
$F(x,y) = (3, 5)$				
$F(x,y) = (y, 0)$				

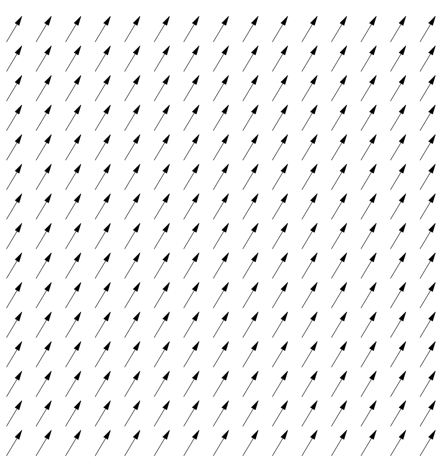
I



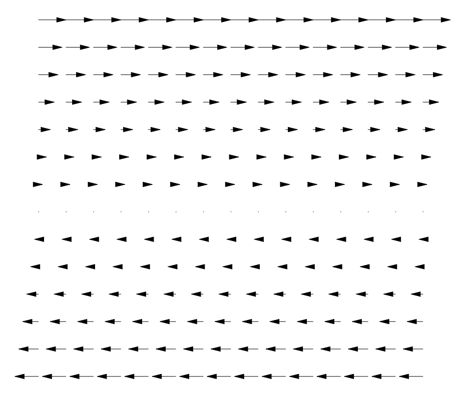
II



III



IV



Problem 3)	(10 points)

Use the technique of linear approximation to estimate $f(\log(2) + 0.001, 2\log(2) + 0.06)$ for $f(x,y) = e^{2x-y}$.

Problem 4) (10 points)

a) Show that for any differentiable function $g(x)$, the function $u(x,y) = g(x^2+y^2)$ satisfies the partial differential equation $yu_x = xu_y$.

b) Assuming $g'(5) \neq 0$, let u be the function defined in a). Find the direction of maximal increase of u at the point $(x,y) = (2,1)$.

Problem 5) (10 points)

Which point on the surface $g(x,y,z) = 1/x + 1/y + 1/z = 1$ is closest to the origin?

Problem 6) (10 points)

Find the maximal and minimal value of the function $f(x,y) = 2x^2 + xy + y^2 - 7x$ on the plane and characterize all extrema.

Problem 7) (10 points)

Find

$$\int_0^1 \int_y^1 x^2 e^{xy} \, dxdy \; .$$

Hint. Sketch the region and check the order of integration.

Problem 8) (10 points)

Integrate the function $f(x,y,z) = z^2$ over the unit ball $\{x^2 + y^2 + z^2 \leq 1\}$.

Problem 9) (10 points)

Find the surface area of the surface $z = x^2 - y^2 + 1$ that lies above the unit disk $x^2 + y^2 \leq 1$