SECOND PRACTICE EXAM FIRST HOURLY **Math 21a,Spring 2003**

Name:

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MWF10 Weiyang Qiu
MWF11 Oliver Knill
TTh10 Mark Lucianovic
TTh11.5 Ciprian Manolescu

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) (20 points)

Circle for each of the 20 questions the correct letter. No justifications are needed. Your score will be $C - W$ where C is the number of correct answers and W is the number of wrong answers.

T	F	The vector connecting the point $(1, 4, 2)$ with the point $(1, 1, 1)$ is parallel to the vector $\langle 0, -6, -2 \rangle$.
T	F	The length of the sum of two vectors is the sum of the length of the vectors.
T	F	For any three vectors, $\vec{v} \cdot (\vec{w} + \vec{u}) = \vec{w} \cdot \vec{v} + \vec{u} \cdot \vec{v}$.
T	F	For any three vectors, $(\vec{v} \times \vec{w}) \cdot \vec{u} = (\vec{w} \times \vec{v}) \cdot \vec{u}$.
T	F	For any three vectors $\|(\vec{u} \times \vec{v}) \cdot \vec{w}\| = \|(\vec{u} \times \vec{w}) \cdot \vec{v}\|$.
T	F	The vectors $\vec{i} + \vec{j}$ and $\vec{k}$ are orthogonal.
T	F	For any vector $\vec{v}$ one has $\vec{v} \times (2\vec{v}) = 0$.
T	F	If we attach the vector $\langle 2, 1, 1 \rangle$ to the point $P = (2, 3, 4)$, the head of the vector points to the point $Q = (3, 4, 5)$.
T	F	The set of points which have distance 1 from a plane form a single plane.
T	F	The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.
T	F	If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then the vectors $\vec{u}$ and $\vec{v}$ have the same length.
T	F	If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R.
T	F	The line $\vec{r}(t) = (1+2t, 1+3t, 1+4t)$ hits the plane $2x+3y+4z = 9$ at a right angle.
T	F	If in rectangular coordinates, a point is given by $(1, -1, 0)$, then its spherical coordinates are $(\rho, \theta, \phi) = (\sqrt{2}, -\pi/2, \pi/2)$.
T	F	If the velocity vector of the curve $\vec{r}(t)$ is never zero and always parallel to a constant vector $\vec{v}$ for all times t, then the curve is a straight line.
T	F	The equation $r = 3z$ in cylindrical coordinates defines a cone.
T	F	There are curves for which the binormal vector is not defined at some points.
T	F	The curvature of a curve is the length of the acceleration vector
T	F	A surface which is given as $r = \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the y axis.
T	F	The identity $\|\vec{v} \cdot \vec{w}\|^2 + \|\vec{v} \times \vec{w}\|^2 = \|\vec{v}\|^2 \|\vec{w}\|^2$ holds for all vectors $\vec{v}, \vec{w}$.

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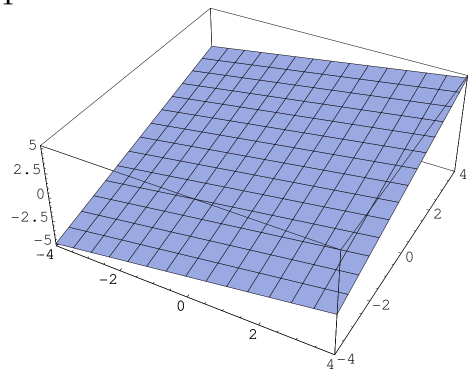
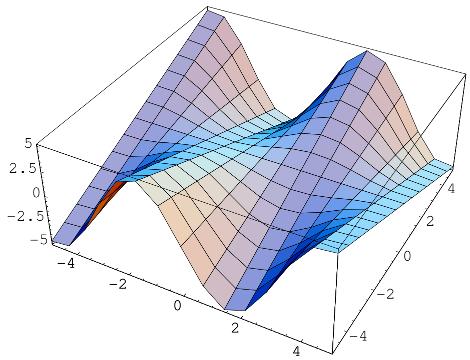
Problem 2) (10 points)

Match the equation with their graphs. To do so, it can help to look at the intersection of each surface with the xy-plane.

I



II



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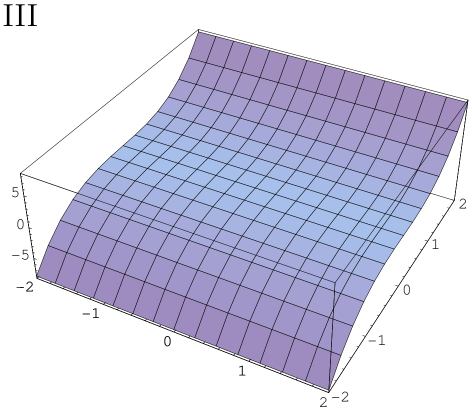

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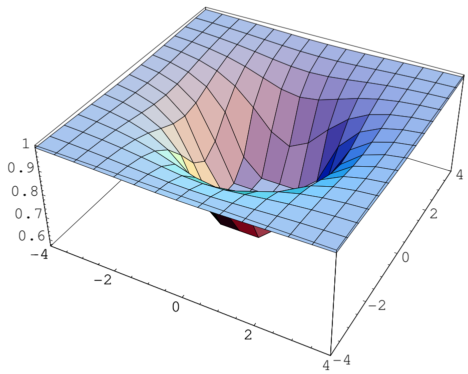

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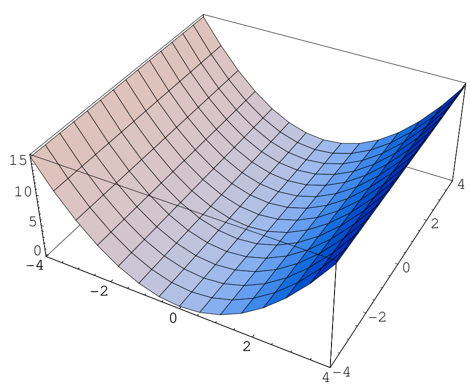

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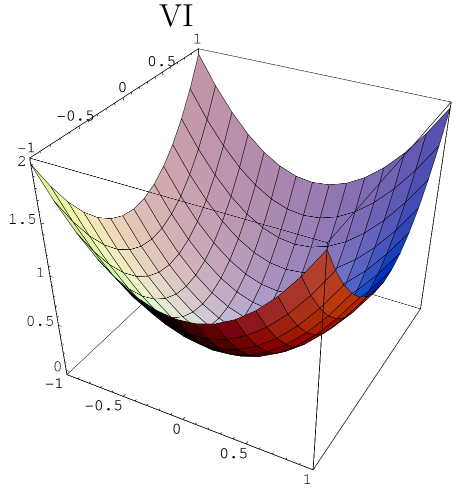


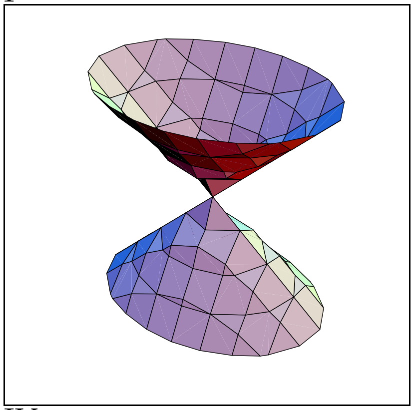
I,II,III,IV,V or VI?	Equation
	$z = \sin(x)y$
	$z = \cos(\frac{\pi}{(1+x^2+y^2)})$
	$2x + 3y + 4z = 0$

I,II,III,IV,V or VI?	Equation
	$z = y^3$
	$z = x^2$
	$z = x^2 + y^2$

Problem 3) (10 points)

Match the equation with their graphs and describe the x-y trace (the intersection of the surface with the xy-plane) with at most three words in each case.

I



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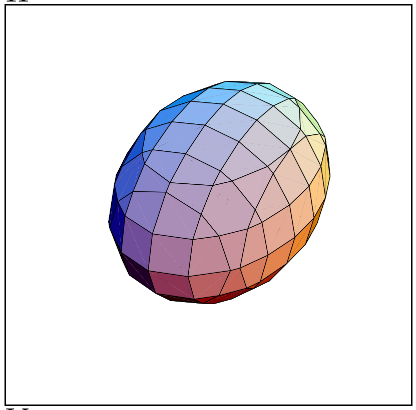
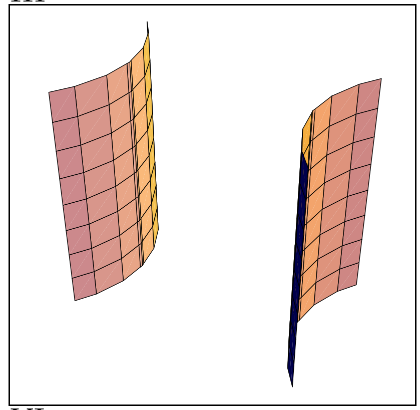
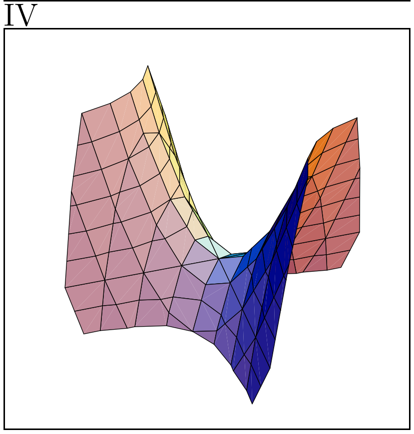


III



IV



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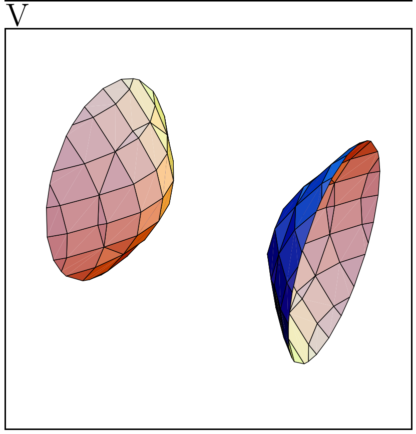
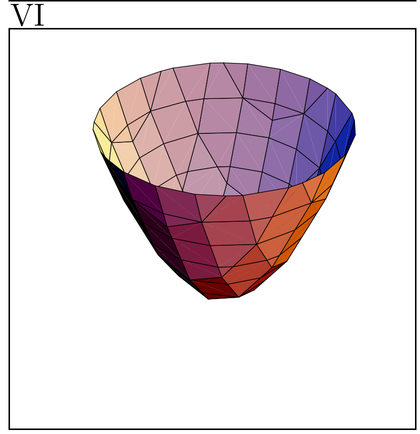


VI



Enter I,II,III,IV,V,VI here	Equation	Describe the x-y trace in words
	$x^2 - y^2 - z^2 = 1$	
	$x^2 + 2y^2 = z^2$	
	$2x^2 + y^2 + 2z^2 = 1$	
	$x^2 - y^2 = 5$	
	$x^2 - y^2 - z = 1$	
	$x^2 + y^2 - z = 1$	

Problem 4) (10 points)

Find the distance between the point $P = (1, 0, -1)$ and the plane which contains the points $A = (1, 1, 1)$ and $B = (0, 2, 1)$ and $C = (1, 2, 2)$.
To do so:

a) Find an equation of the plane.

b) Find the distance.

Problem 5) (10 points)

An ant has gotten into the math department 'surface cabinet' and is walking around on one of the models. Her position in cylindrical coordinates is

$$\begin{aligned} r(t) &= 2\sqrt{t} \\ \theta(t) &= t \\ z(t) &= 2t \end{aligned}$$

for $0 \leq t \leq 6\pi$.

a) What are the parametric equations describing the ant's path in rectangular coordinates?

b) Write an equation (in either rectangular or cylindrical coordinates) which might describe the surface the ant is walking on. Sketch the surface given by your equation, and indicate the ant's path.
(Hint: there are many possible surfaces. You may find some easier to draw than others.)

Problem 6) (10 points)

a) Find the curvature of the curve $\vec{r}(t) = \langle t^2, t^3, 1 \rangle$ at the point $t = 1$.

b) Is there a point $\vec{r}(t)$ with $\vec{r}\,' \neq \vec{0}$ for which the curvature is zero? Explain.

Hint. Use the formula $\kappa(t) = |\vec{r}\,'(t) \times \vec{r}\,''(t)|/|\vec{r}\,'(t)|^3$.

Problem 7) (10 points)

Let $\vec{a}$ and $\vec{b}$ be two vectors in $\mathbf{R}^3$. Assume that the length of $\vec{a} \times \vec{b}$ is equal to 10. What is the length of $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$?

Problem 8) (10 points)

Consider the parametrized curve $\vec{r}(t) = \langle e^t \cos(t), e^t \sin(t), e^t \rangle$.

a) Find a parametric equation for the tangent line to this curve at $t = \pi$.

b) Find a scalar equation for the plane perpendicular to the curve at the same point.

c) Find the arclength of the segment of the curve for which $0 \leq t \leq 1$.

Problem 9) (10 points)

Let C be the curve of the intersection of the elliptical cylinder $\frac{x^2}{25} + \frac{y^2}{9} = 1$ in three dimensions with the plane $3z = 4y$.

a) Find a parametric equation $\vec{r}(t) = (x(t), y(t), z(t))$ of C.

b) Find the arc length of C.